SafeLand

Living with landslide risk in Europe: Assessment,
effects of global change, and risk management strategies

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Deliverable D0.3

Dealing with uncertainties in modelling, prediction, and decision-making

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SUMMARY

This deliverable report provides a review of the state-of-the-art in dealing with uncertainties in modelling, prediction, and decision-making. The different types and sources of uncertainties that are encountered in modelling, prediction and decision making are covered in Chapter 2 of this report. Chapter 3 gives generic guidelines for selecting the appropriate method for the treatment of uncertainties. The guidelines give information on which techniques can be used for the formulation of uncertainty for input parameters and which methods are applicable to propagate the uncertainty from input to output parameters. A brief review of the most relevant techniques and propagation methods is given in Chapter 4. Chapter 5 contains a description of methods to deal with uncertainty in decision making; here, focus in directed on the description of Bayesian decision methods and analysis. Finally, an example on the modelling and management of uncertainties associated with rockfall hazards following a Bayesian approach is provided in Chapter 6.

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CONTENTS

1   INTRODUCTION

1.1   BACKGROUND

Decision making under uncertainty is an essential aspect of risk management – the larger the uncertainty and the closer to critical, the greater the need for evaluating its effect(s) on the results and consequences. Estimates of risk are pervaded by significant uncertainty due to the uncertainty in data and indicators, and uncertainty in models which use data and indicators as inputs. Neglecting uncertainties could lead to an unsafe estimate of loss, thereby hindering the desired reduction of risk to acceptable levels, or to an overestimation of risk, resulting in un-economic mitigation countermeasures.

Traditionally, risk has been treated as a deterministic metric. Over the last years, however, the importance of handling uncertainty has been recognized, and the topic has received more attention.

The purpose of this deliverable is to summarise the methods for treatment of uncertainties in the quantitative assessment of landslide risk, and in the decision-making process for landslide risk management. Uncertainty analysis pertains conceptually to both qualitative and quantitative perspectives and to all factors influencing the risk assessment and management process. Temporal and spatial variability should be addressed consistently in the context of uncertainty analysis.

Key questions related to the treatment of uncertainty in risk assessment and management are:

1. Which metric(s) should be used for risk quantification (expected casualties, monetary units, etc.)?
2. Which indicators/parameters are more important in determining the risk level?
3. How are the indicators described; as qualitative, semi-quantitative or quantitative parameters?
4. Which uncertainty (and possibly: how large?) is associated with each indicator?
5. What kind of models could be used for hazard, vulnerability and loss estimation: implicit or explicit? If explicit: Which models?
6. Which uncertainties (and possibly: how large?) are associated with the hazard, vulnerability and loss model?
7. Which model(s) for propagation of uncertainties could be used?
8. How should the uncertainty in the estimated risk be accounted for in the decision making process?

This deliverable aims to address the above questions. Key elements of the classification structure and model described in this report (Chapters 2, 3 and 4) are adopted from the EU project MOVE, where treatment of uncertainty was analysed for vulnerability assessment (MOVE, 2011).

1.2 TREATMENT AND ASSESSMENT OF UNCERTAINTY IN THE SAFELAND PROJECT

Table 1.1 gives an overview of the key SafeLand work packages and deliverables which addresses uncertainty, either through the development or review of methodologies for the treatment of uncertainty or by assessing uncertainty for landslide relevant data or parameters.

Table 1.1 Listing of deliverables and work packages that explicitly deal with treatment or assessment of uncertainty in SafeLand

No.	Title	Responsible	Delivery month
D0.3	Dealing with uncertainties in modelling, prediction, and decision-making	D0.3: ETHZ	D0.3: 24
WP1.2	Geomechanical analysis of weather-induced triggering processes: • "Analytical and numerical codes for analysis and prediction of rainfall induced landslides at small scale will be set up. This will include model developments and improvements, especially in the constitutive modelling area. The uncertainties and of the reliability of the prediction will be evaluated." Reported in "D1.2 Geomechanical modelling of slope deformation and failure processes driven by climatic factors: Shallow landslides, deep landslides and debris flows" and "D1.4 Guidelines for use of numerical codes for prediction of climate-induced landslides".	WP: AMRA D1.2: AMRA / ETHZ D1.4: EPFL	D1.2: 12 D1.4: 15
WP1.3	• Statistical studies of thresholds for precipitation-induced landslides. Reported in "D1.5 Statistical and empirical models for prediction of precipitation-induced landslides."	WP: ICG D1.5: ICG	D1.5: 12
WP2.2	Vulnerability to landslides. • "The proposed specific fragility functions and damage states for every element at risk will encompass and quantify various sources of uncertainties, related to the temporal probability and the probability of spatial impact of a specific slope failure or rock fall, for every element at risk. There are various sources of uncertainties (natural or random, epistemic, site characterization, mathematical or model and others). The fragility functions that will be proposed will consider most of these uncertainties quantified through probability distribution functions." • "Task 3: Definition of the fragility functions and damage states for every element at risk considering various uncertainties" Reported in "D2.5 Physical vulnerability of elements at risk to landslides:	WP: AUTH D2.5: AUTH	D2.5: 18

	Methodology for evaluation, fragility curves and damage states for buildings and lifelines."		
WP2.3	Development of procedures for QRA at regional scale and European scale • Task 4. Validation of QRA schemes and zoning maps: "The uncertainty in QRA and zoning will be assessed by analysis of the uncertainty of the risk components" Reported in "D2.9 – Toolbox for landslide quantitative risk assessment"	WP: D2.9: UPC	D2.9: 30
WP3.1	Climate change scenarios for selected regions in Europe • "These experiments will allow for a detailed statistical investigation of possible changes in frequency and intensity of future extreme weather events as well as for an estimation of uncertainty in these projected changes." Synthesised in "D3.4 – Report on projected changes in meteorological extreme events in Europe with a focus on Southern Italy, the Alps, Southern Norway, and Romania: synthesis of results."	WP: MPG D3.4: MPG	D3.4: 32
WP3.3	Landslide hazard evolution in Europe and risk evolution in selected "hotspot" areas • In the activity "Propagating uncertainties into new updated hazard and risk maps"..."[a] new method, based on a possibilistic approach will be developed. It will permit the assessment of uncertainties inherent to a scenario modelling approach." Reported in "D3.9 – Methodology for predicting the changes in the landslide risk during the next 100 years at selected sites in Europe. Changing pattern of landslide risk in hotspot and evolution trends Europe according to global change scenarios."	WP: BRGM D3.9: BRGM / ETHZ	D3.9: 32
WP4.1	Short-term weather forecasting for shallow landslide prediction • Task 3 on "Development and testing of models for infiltration and stability in shallow slopes": "Further developments will regard the integration in the model of the soil moisture monitoring data at different scales (from slope parcels, using in situ monitoring instrumentation, to large area using remote sensing data), the improvement of the soil saturation component, statistical treatment of the input parameters to reduce uncertainty and revision of the infinite slope approach." Reported in "D4.2 – Short-term weather forecasting for prediction of triggering of shallow landslides – Methodology, evaluation of technology and validation at selected test sites"	WP: CMCC D4.2: UNIFI-ICL	D4.2: 12
WP5.1	Toolbox for landslide hazard and risk mitigation measures	WP: ICG	

	• "The toolbox will include technical specifications or policy prescriptions (how to), document, with hindsight, the experience and effectiveness of the approach (do's and don'ts), and estimate the costs, benefits, hazards and vulnerability associated with each measure, including uncertainties." • Task 4 on: "The uncertainty-based (i.e. probabilistic) analyses will interact closely with the other WP's. The research will generate probabilistic estimates of landslide risks and analyse the costs and benefits of structural and non-structural mitigation measures. The measures will take into account future land-use and climate change, including spatial and temporal factors. The innovation of this task is to demonstrate a methodology for carrying out a probabilistic (future oriented) cost-benefit analysis of mitigation options. Emphasis will be placed on documenting the uncertainty bounds required for the analyses." Reported in "D5.4 Quantification of uncertainties in the risk assessment and management process"	D5.4: ETHZ/ICG	D5.4: 24

A short summary and extract of the work involving the treatment and analysis of uncertainties in the some of the completed deliverables listed in Table 1.1 is provided below.

In deliverable D1.5 (Statistical and empirical models for prediction of precipitation-induced landslides), a procedure has been proposed for incorporating uncertainty in time (or date) of occurrence in the estimation of thresholds. This has also been applied to a study area in South-Eastern Norway. This procedure allows to incorporate all events in an inventory, regardless of their uncertainty in the time (or date) of occurrence. The procedure has significance when it is assumed that the time uncertainty is inversely proportional to the magnitude of the landslide.

Deliverable D2.5 has reported on the work carried out on the proposition and quantification, in a measurable and reproducible way, of efficient methodologies for assessing the physical vulnerability of buildings (or sets of buildings) and lifelines exposed to different landslide hazards. The applicability of the developed methodologies varies in relation to the landslide type, specified elements at risk and the analysis scale and the triggering mechanism. The structural vulnerability of the affected facilities is estimated using the concept of probabilistic fragility functions and appropriate definition of relevant damage states including various sources of uncertainty. The determination of an appropriate statistical distribution is of major importance to account for the various sources of uncertainty.

Deliverable D4.2 looks at short-term weather forecasting for prediction of triggering of shallow landslides; the methodology, evaluation of technology and validation at selected test sites are described. Three principal motivations for the uncertainty of the numerical weather forecast have been identified:

- <u>Analysis error</u>: errors in the background fields, observation data and data assimilation techniques used.
- <u>Model uncertainty</u>: inadequacy of physical model processes
- <u>Atmosphere chaotic nature</u>: the atmospheric motions follow non-linear dynamic, small errors in the analysis may quickly be amplified. This is commonly referred to as the "butterfly effect".

In order to quantify the underlying uncertainties, the probabilistic forecasts generating more predictions beginning from very similar initial states are defined. The generated predictions are usually sorted into groups (clusters); depending from the number of prediction that fall in the same clusters it is possible to associate a probability of occurrence to a certain forecast (Ensemble Prediction System).

It is stated that different approaches exist for the problems of numerical weather prediction:

- <u>Deterministic approach</u>: it postulates that, at least over a certain time period, the laws of physics, as applied to the atmosphere, cam be solved (integrated forward in time) to find the forecast fields given initial data describing the current conditions.

- <u>Probabilistic approach</u>: it is based on the idea of starting a set of forecast integrations from slightly different initial conditions, reflecting the range of uncertainty in the estimated initial state. This ensemble approach allows a probability to be assigned to the likelihood of rainfall (for example).

Further specific details on these as well as the other deliverables looking at uncertainty issues in the SafeLand project can be found in the respective deliverables.

1.3   STRUCTURE OF THIS DELIVERABLE

The different types and sources of uncertainties that are encountered in modelling, prediction and decision making are covered in Chapter 2 of this report. Chapter 3 gives generic guidelines for selecting the appropriate method for the treatment of uncertainties. The guidelines give information on which techniques can be used for the formulation of uncertainty for input parameters and which methods are applicable to propagate the uncertainty from input to output parameters. A brief review of the most relevant techniques and propagation methods is given in Chapter 4. Chapter 5 contains a description of methods to deal with uncertainty in decision making; here, focus in directed on the description of Bayesian decision methods and analysis. Finally, an example on the modelling and management of uncertainties associated with rockfall hazards following a Bayesian approach is provided in Chapter 6.

2   SOURCES AND TYPES OF UNCERTAINTIES

Uncertainty can be analysed and categorised in many different ways (Apostolakis, 1990, Helton and Burmaster, 1996). One possible categorisation is to classify the uncertainty into aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty parameterizes the inherent, "real" variability of the physical environment and represents the natural randomness of a variable. Examples of aleatory uncertainty are the spatial variation of a soil parameter within a nominally uniform geological layer, the temporal variation in the peak acceleration of a design earthquake with a given return period, the variation in the ocean wave height or wind force, and so on. The aleatory uncertainty, which is also called the inherent uncertainty, cannot be reduced or eliminated. Epistemic uncertainty, on the other hand, represents the uncertainty due to lack of knowledge on a variable. Epistemic uncertainty includes measurement uncertainty, statistical uncertainty (due to limited information), and model uncertainty. Statistical uncertainty is due to limited information such as limited number of observations. Measurement uncertainty is due to for example imperfections of an instrument or of a method to register a quantity. Model uncertainty is due to idealizations made in the physical formulation of the problem. Epistemic uncertainty is "artificial" and can be reduced, perhaps even eliminated, by collecting more data and information, improving the measurement method(s) or improving the calculation method(s).

A second possible categorisation refers to the method of uncertainty modelling. Objective quantification of uncertainty is based on processing (e.g. by statistical and probabilistic methods) of available data for indicators. Subjective modelling relies on the analyst's experience (expert judgement), prior information, belief, necessity or, more frequently, a combination thereof.

A third possible categorisation of uncertainties refers to at which stage in the risk estimation process they are located, i.e. in the input parameters to the models (parameter uncertainty) or in the models (transformation uncertainty) which in turn determine the uncertainty of the output parameters. In general, parameter uncertainty is partly aleatory and partly epistemic. Transformation uncertainty is due to the approximations and simplifications inherent in empirical, semi-empirical, experimental or theoretical models used to relate model inputs to model outputs. It is essentially epistemic in nature.

In Figure 2.1 the classification system from MOVE (2011) is shown. It is based on the third categorisation above. A risk assessment method will be characterised by the path it follows through the three process stages defined in Figure 2.1: (1) inputs, (2) models/procedures and (3) outputs. The path followed may be iterative. Many explicit methods contains for example data that have been derived through implicit methods/expert judgement.

Inputs can be classified as qualitative, categorical or quantitative according to the following description:
- Qualitative: magnitude of parameters described verbally; no numerical categorical or quantitative value associated with verbal description
- Categorical: magnitude of parameters expressed on a quantitative, purposely defined ordinal scale
- Quantitative: magnitude of parameters expressed on a quantitative, measurable scale

Models / procedures can be classified as implicit or explicit according to the following description:

- Implicit: Aggregation of inputs occurs subjectively, based on expert judgment
- Explicit: Aggregation of inputs relies on repeatable criteria, algorithms, models or formulae

Outputs can be classified as qualitative, categorical or quantitative according to the following description:

- Qualitative: magnitude of outputs described verbally; no numerical categorical or quantitative value associated with verbal description
- Categorical: outputs are expressed on a quantitative, purposely defined ordinal scale
- Quantitative: magnitude of outputs is expressed on a quantitative scale

Figure 2.1 Proposed classification system for treatment of uncertainty in risk estimation (Source: MOVE, 2011).

The type of output data depend on the type of input data and choice of method. For instance a methodology may be based on quantitative inputs, but through the use of an explicit

procedure end up with categorical or semi-quantitative outputs. Another approach may be based on qualitative inputs but through an implicit procedure, quantitative outputs can be produced.

The appropriate method for treatment of uncertainty will depend on which path the risk estimation method takes through the diagram in Figure 2.1, i.e. which combination of data type and method.

3   FORMULATION OF UNCERTAINTY IN INPUT PARAMETERS

This chapter briefly describes the various methods for formulating uncertainty in terms of qualitative and quantitative parameters. The presentation is adapted from MOVE (2011).

3.1   QUALITATIVE PARAMETERS

Qualitative parameters could be easily distinguished from quantitative parameters by arguing that every parameter that could be expressed as a number or value is a quantitative parameter everything else is qualitative (Stegmüller 1970). In order to obtain relevant qualitative input parameters a systematic measurement method is needed. Therefore transforming qualitative parameters to semi-quantitative parameters with nominal scales such as yes/no, good/bad or a ranking scale could be very useful. Although qualitative parameters are used as the outcome of the nominal scale these could still be analyzed quantitatively (Mayring 2007).These qualitative parameters could be derived through expert judgment, elicited either through individual interviews, interactive groups, or Delphi situations.

3.1.1   Semi-quantitative data - Possibility theory

Transforming a quantitative parameter into a semi-quantitative parameter may be useful in some cases. Instead of expressing risk in terms of probabilities, formulating the results from a risk assessment into categories, for instance "low", "medium" and "high" risk may ease communication and help decision makers prioritize and target risk mitigation measures. Another situation where categorization of quantitative parameters may be appropriate is when significant uncertainty is associated with the quantitative parameter and expressing the parameter with a precise numerical value may convey a false message of high precision. Such uncertainty may stem from incomplete knowledge and imprecision due to lack of information resulting, for example, from systematic measurement errors or expert opinions (Baudrit and Dubois, 2006).

To transform a quantitative parameter into a semi-quantitative/categorical parameter, possibility theory can be applied. An expert commonly estimates the numerical values of a parameter using confidence intervals according to his/her experience and intuition. The interval is defined by a lower and an upper bound. In most cases, experts may provide more information by expressing preferences inside this interval. For example, "an expert is certain that the value for the model parameter is located within the interval [a, b]". However, according to a measurements and experience, the expert may be able to judge that "the value for the model parameter is most likely to be within a narrower interval [c, d]". To represent such information, an appropriate tool is the possibility distribution, based on the Fuzzy set theory (Zadeh 1965), which describes the more or less plausible values of some uncertain quantity (Dubois and Prade 1988). The preference of the expert is modelled by a degree of possibility (*i.e.* likelihood) ranging from 0 to 1. In practice, the most likely interval [c, d] (referred to as the "core") is assigned a degree of possibility equal to one, whereas the "certain" interval [a, b] (referred to as the "support") is assigned a degree of possibility zero, such that values located outside this interval are considered impossible. Figure 3.1 illustrates such an approach in the field of earthquake hazard assessment.
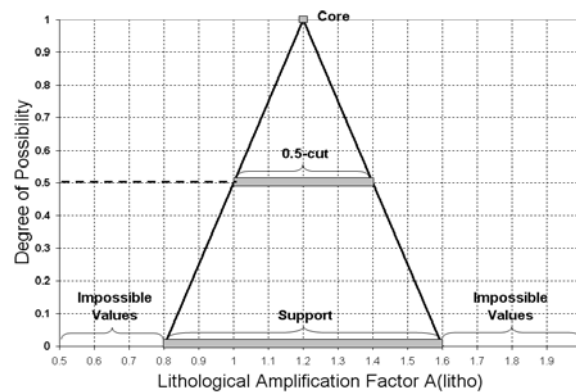
Figure 3.1 Definition of the possibility distribution representing the uncertainty of the lithological amplification factor for the seismic hazard assessment at the local scale (i.e. site effect). The « Core » is {1.2}, The Support is [0.8-1.6] and the confidence interval at 50 % is represented by the 0.5-cut (adapted from Rohmer, 2007).

3.2   QUANTITATIVE PARAMETERS

Quantitative parameters can be expressed as an expected value, together with some measure of the uncertainty/spread of the data (for example standard deviation or coefficient of variation) or as a full probability density function. To select a distribution type for the probability density function, the properties of samples of a random variable must be consistent with the definition and attributes of the variable itself. The selection of a probability distribution to suitably represent a random variable can be made, for instance, using the *principle of maximum entropy*.

Table 3.1 provides an example of the various criteria and constraints which may be used in selecting a probability distribution using the maximum entropy principle. For example, if only the mean and the standard deviation of a random variable are known, and negative values are not acceptable even for small probability levels, a lognormal distribution should be selected. If, for instance, the minimum, maximum, mean value and standard deviation of a random variable are known, the probability distribution to be adopted is a Pearson type-I beta distribution.

Table 3.1 Maximum-entropy criteria and constraints for the selection of a probability distribution (Source: MOVE, 2011)

mean	mode	variance	negative values	Lower bound	Upper bound	maximum entropy distribution
N	N	N	Y	Y	Y	Uniform
Y	N	Y	Y	N	N	Normal
Y	N	Y	N	Y	N	Lognormal
Y	N	Y	Y	Y	Y	type-I beta
N	Y	N	Y	Y	Y	PERT, triangular

4   TREATMENT OF PROPAGATION OF UNCERTAINTIES

When using input data associated with uncertainties in a risk assessment model (either implicit or explicit as described in Figure 2.1) the uncertainties propagate through the model and influence the output. There are several different methods for characterizing or quantifying the effect of uncertainties in the input on the resulting expected value and uncertainties in the output. This chapter is partly adapted from MOVE (2011) and summarizes the most used methods.

Some of the uncertainty propagation methods described in this section are flexible in terms of which type of input data they can handle; either qualitative, semi-quantitative or quantitative data. However, most methods are designed to process certain types of data. Expert judgment and Qualitative scenario analysis are mostly used for qualitative data. The NUSAP (Numerical, Unit, Spread, Assessment and Pedigree) method, Hybrid methods and Fuzzy logic are often used to treat semi-quantitative data. Bayesian theory, Monte Carlo Simulation (MCS), First Order Second Moment approximation (FOSM), Point Estimation Methods (PEM) and First Order Reliability Method (FORM) are most often used in connection with quantitative data.

4.1   IMPLICIT METHODS

4.1.1   Expert judgment

Expert judgments are the expressions of informed opinion, based on knowledge and experience that experts make in responding to technical problems (Booker & Meyer, 2001). Booker & Meyer (2001) define an expert as an individual who has background in the subject area and is recognized by peers as having the necessary qualifications to address the technical problem. Judgements, according to Otway and Winterfeld (1992), are inferences or evaluations that go beyond obvious statements of fact, data, or the conventions of a discipline. Instead, factual judgments are beliefs or opinions about propositions that can, in principle, be proven right or wrong. Value judgments are expressions of preferences between alternatives, based on tradeoffs and priorities. The process of gathering information from experts is expert elicitation. It is a structured process to elicit subjective judgments and ideas from experts. It seeks to make explicit and utilizable the unpublished knowledge and wisdom in the heads of experts, based on their experience and expertise, including their insight in the limitations, strengths and weaknesses of the published knowledge and available data (Refsgaard et al 2006).

Expert judgement can be the result either of informal or formal processes. Informal processes are implicit, unstructured and undocumented. According to Otway and Winterfeld (1992), expert judgement processes have always entered analysis through the experts' implicit and undocumented reasoning, inferences, scientific conventions or even unconscious processes. Examples for such informal judgements are about the identification of:
- the problems that need to be analysed;
- the kind of models should be used;
- the kind of data should be used;
- how should results be interpreted;

- the actions to be recommended.

In contrast, formal uses of Expert judgment are explicit and documented. They involve a deliberate attempt to bring out the assumptions and reasoning underlying a judgment, to quantify it to the extent useful and to document it so that it can be appraised by others. The following steps describe the process of formal Expert judgment elicitation (adapted from Otway and Winterfeld, 1992):

1. identify and select the events and options about which fact or value judgements should be made formally;
2. identify and select the experts who will make the judgments;
3. define the issues for which judgments are to be elicited and identify the relevant experts in the fields;
4. train the experts to the rules of the methodology for the elicitation of formal judgments (i.e. elicitation methods, decomposition approaches, bias identification and debiasing technique);
5. elicit the Expert judgments in interviews with a trained facilitator, who poses questions to properly separate the main problem in sub-problems to be discussed, to elicit judgments (through for instance a brainstorming session), and to cross-check result against other forms of judgement;
6. analyse and aggregate results obtained from individual experts and, in the case of substantial disagreements, attempt to resolve differences;
7. completely document results, including the reasoning given by the experts to support their judgement.

In addition, expert judgement can be elicited simply through one-on-one, individual interviews, or through the use of interactive groups where experts and a moderator meet face to face to give and discuss their data in either a structured or informal manner. In order to avoid the possible negative effects of group interaction (biasing effects such as when the presence of a dominant expert causes others in the group to agree to a judgement they do not hold), the Delphi technique can be applied to minimize bias and maintain anonymity. Under this process a group of experts, in isolation from each other, give their judgements to the moderator. The moderator makes these judgements anonymous, collates them, then and redistributes them to the experts in order for them to revise their previous judgement. This process can be repeated until consensus (if it is required) or a more refined set of data is achieved. The continuously iterative process reduces the risk of uncertainties while limiting inter-expert bias.

Furthermore, according to Booker & Meyer (2001) Expert judgment can be used for statistical applications such as:

- the probability of an occurrence of an event;
- a prediction of the performance of some product or process;
- the decision about what variables enter into a statistical analysis;
- the decision about which data sets to include in an analysis;
- the assumptions used in selecting a model;

- the decision concerning which probability distributions are appropriate to use;
- a description of experts' thinking and information sources in arriving at any of the above responses.

Expert judgment can be expressed in three forms:

- *quantitative form—probabilities* such as ratings, odds, uncertainty estimates, weighting factors, and physical quantities of interest (e.g., costs, time, length, weight, etc.),
- *semi-quantitative form—categorisation/classification* of expert judgment opinion/beliefs or
- *qualitative form—a textual description* of the expert's assumptions in reaching an estimate, reasons for selecting or eliminating certain data or information from analysis, and natural language statements of physical quantities of interest.

There are also named some drawbacks of the expert judgement method. The Expert judgment itself has uncertainty. However, this can be characterized and subsequently analyzed.

The major problem in Expert elicitation is expert bias. Expert judgement is affected by the process of gathering it. Experts and lay people alike are subject to a variety of potential mental errors or shortcomings caused by man's simplified and partly subconscious information processing strategies. It is important to distinguish these so-called cognitive biases from other sources of bias, such as cultural bias, organizational bias, or bias resulting from one's own self-interest (Heuer, 1999). Some of the sources of cognitive bias are as follows: overconfidence, anchoring, availability, representativeness, satisficing, unstated assumptions, coherence. Experts should be informed on the existence of these biases during the Expert elicitation process. Gigerenzer (1991,1994) and Cosmides and Tooby (1996) argue that part of these biases are not so much caused by the limited cognitive abilities of the human mind, but more by the way in which information is presented or elicited. A thoughtful wording of questions can be helpful to avoid part of these biases. Performing dry run exercises (try-outs) can render important feedback on the suitability of the posed questions.

Many experts are accustomed to giving uncertainty estimates in the form of simple ranges of values. In eliciting uncertainties, the analysts can make experts aware of their natural tendency to underestimate uncertainty, such as through the exercise of estimating on sample problems. Studies have shown that experts are typically unable to completely overcome this tendency (Booker & Meyer, 2001). Several elicitation protocols have been developed whereas the Stanford/SRI Protocol is the most commonly used. For detailed information please see Spetzler and von Holstein (1975), Merkhofer (1987) and Morgan and Henrion (1990). Another protocol which can be used is from Cooke & Goossens (2000 a, b), adapted by Van der Fels-Klerx et al (2002). The authors of the reviewed literature emphasised that there is not a preferable method, and in practise it is recommended to use a mix of both methods, depending on the availability and quality of data and information.

In addition, to perform a formal Expert elicitation is a time and resource intensive activity. The whole process of setting up a study, selecting experts, preparing elicitation questions,

expert training, expert meetings, interviews, analyses, writing rationales, documentation (as described above) can easily stretch over months or years.

Furthermore should be pointed out that the Expert judgement can be viewed as a snapshot of the expert's state of knowledge at a specific time; hence this judgement could change over time due to the fact that the expert could receive new information. In addition, because the judgment reflects the expert's knowledge and learning, the experts can validly differ in their judgments.

4.1.2   Qualitative scenarios analysis

There exist in the literature many definitions of what a scenario is. A general definition is: "scenarios are descriptions of possible futures that reflect different perspectives on the past, the present and the future (van-Notten & Rotmans 2001) cited in (van-Notten et al 2003)". In the context of sustainability science, scenarios are coherent and plausible stories, told in words and numbers, about the possible co-evolutionary pathways of combined human and environmental systems (Swart et al 2004). Scenarios generally include: a definition of problem boundaries, a characterisation of current conditions and drivers of change and the identification of uncertainties (Swart et al 2004).

Scenario analysis is a common approach used when only partial information, or no information at all, is available. Thus it is meant to deal with uncertainty and support strategic decision making. The uncertainty about parameters or components of the system is addressed through the description of a small number of "sub-problems" derived from an underlying optimization problem. The idea is that, by describing and studying the different "sub-problems" and their optimal solutions, similarities and trends may eventually be identified and uncertainty reduced. An optimal solution to the underlying problem may then be suggested. Qualitative scenario analysis is used to stimulate brainstorming, through experts workshops, about an issue, when many views about the future have to be included or when an idea has to be formed about, for example, general social and cultural trends (Sluijs et al 2004). Hodgson (1992) proposed the "scenario-thinking" concept, which is useful for gathering a wide variety of perspectives from actors and develop storylines. It makes use of visual facilitation tools, in particular hexagons, as a flexible mapping technique to bridge the gap between thoughts and models (Giupponi et al 2006). Other type of scenarios are benchmark scenarios, policy scenarios or exploratory, anticipatory scenarios (Sluijs et al 2004). Quantitative scenario analysis is also used to for assessments that require data and numbers (Sluijs et al 2004).

4.1.3   Numerical, unit, Spread, assessment and pedigree (NUSAP)

NUSAP is a notational system proposed by Funtowicz and Ravetz (1990), which aims to provide an analysis and diagnosis of uncertainty in science for policy. It captures both quantitative and qualitative dimensions of uncertainty and enables one to display these in a standardized and self-explanatory way. It promotes criticism by clients and users of all sorts, experts as well as lay public, and will thereby support extended peer review processes.

The NUSAP method is based on five categories, which generally reflect the standard practice of experimental sciences. By providing a separate box, or "field", for each aspect of the information, it enables a great flexibility in their expression. The name "NUSAP" is an acronym for the categories. The first, "Numeral", will usually be an ordinary number; but when appropriate it can be a more general quantity, such as the expression "a million" (which is not the same as the number lying between 999,999 and 1,000,001). The second is "Unit", which may be of the conventional sort, but which may also contain extra information, as the date at which the unit is evaluated. The third category is "Spread", which generalizes from the "random error" of experiments or the "variance" of statistics. Although "Spread" is usually expressed by a number (either $\pm$, % or "factor of") it is not an ordinary quantity, for its own inexactness is not of the same sort as that of measurements. The next letter A for "Assessment" leads us to the more qualitative side of the NUSAP method where the Expert elicitation is an appropriate method for. Finally the P for "Pedigree", it expresses an evaluation account of the production process of information, and indicates different aspects of the underpinning and scientific status of knowledge used (Sluijs et al 2004). In order to assess these different aspects a set of pedigree criteria is used like for example proxy representation, empirical basis, methodological rigor, theoretical understanding and validation. The assessment of pedigree also includes qualitative Experts judgment. To minimise arbitrariness and subjectivity in measuring strength, a pedigree matrix is used to code qualitative Expert judgements for each criterion into a discrete numeral scale from 0 (weak) to 4 (strong) with linguistic descriptions (modes) of each level on the scale (Refsggaard et al., 2006).

4.1.4   Fuzzy logic

Fuzzy logic was introduced by Lofti Zadeh in 1965. The method concentrates on solving specific problems rather than trying to model a whole system. It is a multi-valued logic, which introduces fuzzy sets in addition to crisp sets. In crisp sets elements either belong to a set or not. Fuzzy sets allow elements to belong to a set to a certain degree. Therefore fuzzy logic allows one to integrate imprecise approximations and semantic notions such as high / medium / low into mathematical formulations and computer models.

The first step in a fuzzy logic system is called fuzzyfication: for each input parameter a degree of membership is assigned for linguistic values by the membership function of the fuzzy set. Based on expert knowledge the second step connects the fuzzy set with logic rules for the linguistic values. Also preconditions of the fuzzy sets have to be applied to the linguistic values depending on the actions of the logic rules. This is called inference. In most cases there is more than one logic rule applied to linguistic values in rule systems. Therefore membership functions of the actions have to be combined after inference building into a generic membership function. This is called composition. Table 4.1 shows the logic rules for the example of SAR remote sensing data by Hellmann (2001). The last step in a fuzzy rule system is called defuzzyfication. It derives an output parameter from the generic membership function Reif (2000). Figure 4.1 shows an example by Hellmann (2001) for the classification of SAR remote Sensing data.
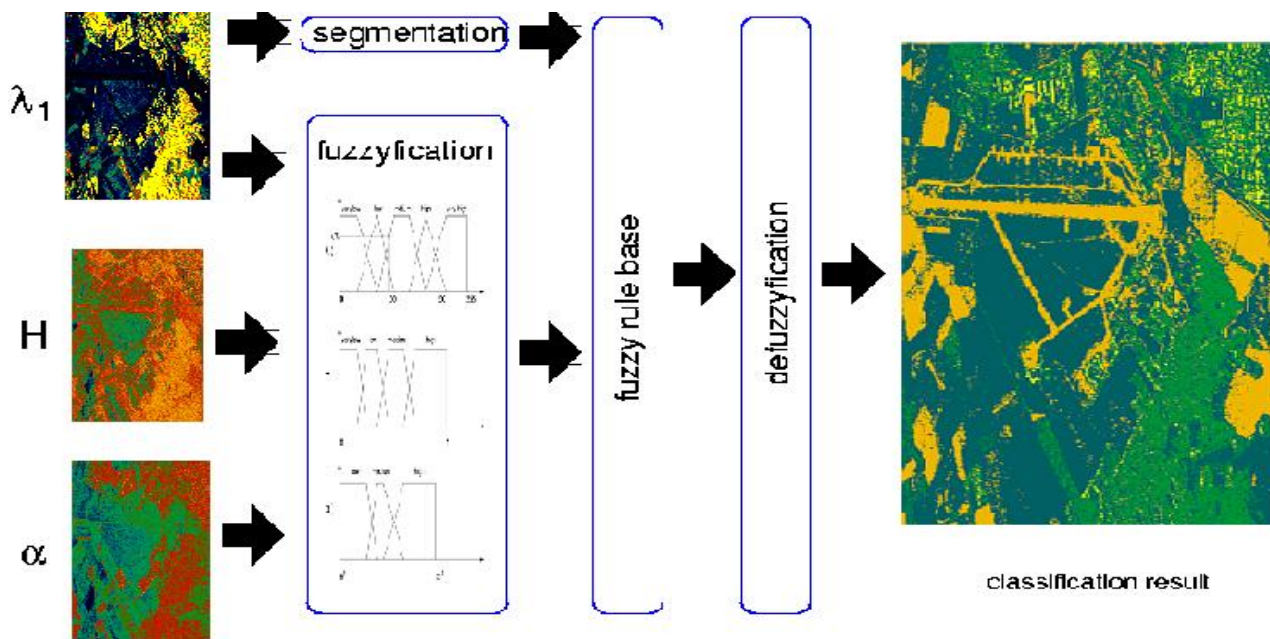
Figure 4.1 Fuzzy classification scheme for an SAR remote Sensing data example by Hellmann (2001).

Table 4.1 Classification rules for a SAR remote sensing data example by Hellmann (2001)

λ_1	H	α	*Decision*
very high	medium		urban
high or very high	very low	medium/high	urban
high	high		forest
medium	high	medium/high	forest
medium	medium	medium/low	vegetation
medium	low or very low	low	vegetation
(very) low			runway

In general, the employment of fuzzy logic might be helpful for very complex processes, when there is missing data or high uncertainties, when no simple mathematical model exists, for highly nonlinear processes or for the processing of linguistically formulated expert knowledge.

The difficulty of the fuzzy approach lies in the requirement of sufficient expert knowledge for the combination of the sets, the combination of the fuzzy rule base and the defuzzyfication. Fuzzy logic is not recommendable, if expert knowledge is lacking, a conventional approach yields a satisfying result or an alternative easily solvable and adequate mathematical model already exists; Hellmann (2001).

4.2   EXPLICIT METHODS

4.2.1   Monte Carlo simulation (MCS)

The term "Monte Carlo simulation" (MCS) embraces a wide class of computational algorithms which are effectively capable of simulating complex physical and mathematical systems. The simulations are performed by repeated deterministic computation of user-defined transformation models using random values as input (i.e. random values drawn from user-generated sampling distributions). MCS is often used when the model is complex, nonlinear, or involves several uncertain parameters. The number of evaluations/repetitions necessary to establish the probability distributions of the output parameters will depend on the number of input parameters, their probability distributions, the complexity of the propagation model and the accuracy requirements of the output. A simulation can easily involve over 10,000 evaluations of the model.

Monte Carlo simulation allows consistent processing of uncertainties in input parameters and models, regardless of the degree of linearity and the complexity of transformation models, and of the magnitude of uncertainties in parameters and models. Other notable advantages of MCS over other techniques include: (a) the possibility of appreciating (to the degree of precision desired by the user and imposed by the quality of input data) the shape of the output variable; (b) the possibility of including complex mathematics (e.g. logical statements) with no extra difficulty; and (c) the possibility to model transformation uncertainty directly and to assess its effect on model outputs (Vose 2008).

Monte Carlo simulation requires the generation of artificial samples of input random variables from purposely selected distributions. Such process requires sequentially: (a) the assignment of a probability distribution type; (b) the assignment of characteristic distribution parameters; and (c) sampling from the distributions. In addition, computation power needs may be large for complex models.

The Monte Carlo simulation technique is implemented in some commercial slope stability analysis packages (e.g. Geo-Studio, 2010). However, when the probability of failure is very small, the number of simulations required to obtain an accurate result directly is so large that, except for very simple (or simplified) problems, it renders the application impractical. In these situations the conditional probability of failure can be determined for various low probability scenarios, and then combined, considering the scenario probabilities. Monte Carlo simulation can be optimized by stratified sampling techniques, for example Latin Hypercube sampling (Iman & Conover 1982). These "organized" sampling techniques considerably reduce the number of simulations required for a reliable distribution of the response.

4.2.2   Point estimation methods (PEM)

Point estimation methods (PEM), originally proposed by Rosenblueth (1975) and subsequently extended by various researchers (e.g. Hong 1998), are a class of simple and direct procedures for computing low-order moments of functions of random variables. However, they suffer from several shortcomings. First, they do not perform satisfactorily

when the distribution of the output is significantly different from those of the input variables. Second, PEM methods should not be applied when the transformation model cannot be approximated satisfactorily by a third-order polynomial, and, similarly to FOSM, in presence of large uncertainties in inputs. Another shortcoming of PEM lies in the fact that point estimates are less reliable for statistical moments beyond second-order (Baecher & Christian 2003).

4.2.3 First order, second moment approximation (FOSM)

The basis of FOSM approximation (e.g. Ayyub & McCuen 2003) lies in the statement that satisfactory estimates of the second-moment parameters (mean, standard deviation) of a random variable which is a function of other random variables may be calculated if second-moment parameters of the input random variables and the transformation model relating inputs to output are known. The FOSM approximation uses a Taylor series expansion of the variable to be evaluated.

Consider Y to be a function of random variables $x_1, x_2, ..., x_n$; that is

$$Y = f(x_1, x_2, ..., x_n) \qquad\qquad (4.1)$$

In the general case, $x_1, x_2, ..., x_n$ are correlated with covariance matrix [C], i.e. [C] = [σ][R][σ], where [σ] is a diagonal matrix of standard deviations and [R] is the (positive-definite and symmetric) correlation matrix with diagonal elements $R_{ii} = 1$ and non-diagonal elements $R_{ij} = \rho_{ij}$ (ρ_{ij} is the correlation coefficient between variables i and j). In scalar notation, $C_{ij} = \sigma_i \sigma_j R_{ij}$.

Obviously to evaluate the mean and standard deviation of Y, the joint probability density function of $x_1, x_2, ..., x_n$ is needed. However, in many practical applications the available information about the random variables is limited to their mean and variance. The approximate mean and variance of the function Y may still be estimated by a Taylor series expansion of the function about the mean values of the random variables and neglecting the higher order terms (Ang and Tang, 1984). If the Taylor series is truncated at its linear terms, the following first-order estimates of mean and variance are obtained:

$$\mu_Y \approx f(\mu_{x_1}, \mu_{x_2}, ..., \mu_{x_n}) \qquad\qquad (4.2)$$

$$\sigma_Y^2 \approx \{b\}^T [C]\{b\} \qquad\qquad (4.3)$$

where the vector $\{b\}$ denotes $\partial Y / \partial x_i$ evaluated at the mean values of x_i, i.e.:

$$\{b\}^T = \left\{ \partial Y / \partial x_1, \, \partial Y / \partial x_2 \, ,........, \, \partial Y / \partial x_n \right\}\Big|_{\mu_x}$$

If there is no correlation among the variables, Equation 4.3 can be written as:

$$\sigma_Y^2 \approx \sum_{i=1}^{n} \left(\frac{\partial Y}{\partial x_i}\bigg|_{\mu_{x_i}} \right)^2 \cdot \sigma_{x_i}^2 \qquad\qquad (4.4)$$

Equations 4.2 and 4.3 or 4.4 are referred to as the first-order, second-moment (FOSM) approximations of mean and variance of Y.

The FOSM approximation only provides estimates of the mean and standard deviation, which are not sufficient by themselves for evaluating the failure probability. To estimate the failure probability, one must assume the distribution function for the safety margin or the safety factor beforehand. The first step in estimation of failure probability using any probabilistic method is to decide on what constitutes unsatisfactory performance or failure. Mathematically, this is achieved by defining a performance function G(X), such that $G(X) \geq 0$ means satisfactory performance and $G(X) < 0$ means unsatisfactory performance or "failure". X is a vector of basic random variables including resistance parameters, load effects, geometry parameters and model uncertainty.

Example 4.1

Consider a structural element with resistance R, subjected to dead load D and live load L. The safety margin (performance function) for this element is defined as:

$$G = R - D - L$$

Given the information below, estimate the mean and coefficient of variation (CoV = σ/μ) of G with and without correlation among the parameters.

Mean values: $\mu_R = 2.8$ $\mu_D = 1$ $\mu_L = 0.75$

Standard deviations: $\sigma_R = 0.3$ $\sigma_D = 0.1$ $\sigma_L = 0.2$

Correlation coefficients: $\rho_{R,D} = 0.8$ $\rho_{D,L} = 0.3$

FOSM approximation:

$\mu_G = 2.8 - 1 - 0.75 = 1.05$

$\{b\}^T = \{\partial G / \partial R, \partial G / \partial D, \partial G / \partial L\} = \{1 \ \text{-1} \ \text{-1}\}$

No correlation:

$$\sigma_G^2 = \{1 \quad -1 \quad -1\} \begin{bmatrix} 0.09 & & \\ & 0.01 & \\ & & 0.04 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \\ -1 \end{Bmatrix} = 0.14$$

$\Rightarrow \sigma_G = 0.374$, CoV = 0.374/1.05 = 35.6 %

With correlation:

$$\sigma_G^2 = \{1 \quad -1 \quad -1\} \begin{bmatrix} 0.09 & 0.024 & 0 \\ 0.024 & 0.01 & 0.006 \\ 0 & 0.006 & 0.04 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \\ -1 \end{Bmatrix} = 0.104$$

$$\Rightarrow \sigma_G = 0.323, \text{CoV} = 0.323/1.05 = 30.7\ \%$$

The "reliability index", defined as

$$\beta = \frac{\mu_G}{\sigma_G} \tag{4.5}$$

in which μ_G and σ_G are respectively the mean and standard deviation of the performance function, is often used as an alternative performance measure to the factor of safety (Li & Lumb 1987, Christian *et al.* 1994, Duncan 2000).

The reliability index provides more information about the reliability of a geotechnical design or a geotechnical structure than is obtained from the factor of safety alone. It is directly related to the probability of failure and the computational procedures used to evaluate the reliability index reveal which parameters contribute most to the uncertainty in the factor of safety. This is useful information that can guide the engineer in further investigations. However, the reliability index estimated using the FOSM approach is not "invariant". Table 4.2 shows the reliability indices for different formats of the performance function using the FOSM method. *R* and *S* in the table represent respectively the total resisting force and the driving force acting on the slope. CoV_R and CoV_S in the table denote the coefficients of variation of the resisting and the loading forces respectively and $F = \mu_R / \mu_S$.

Table 4.2 Performance function format and FOSM reliability index β (Li & Lumb 1987).

G(X)	β
$R - S$	$\dfrac{F - 1}{\sqrt{F^2 CoV_R^2 + CoV_S^2}}$
$\dfrac{R}{S} - 1$	$\dfrac{F - 1}{F\sqrt{CoV_R^2 + CoV_S^2}}$
$\ln \dfrac{R}{S}$	$\dfrac{\ln F}{\sqrt{CoV_R^2 + CoV_S^2}}$

While the formulation of FOSM approximation is concise and allows direct inclusion of correlation among input variables, several potential shortcomings should be acknowledged. First, when transformation models are non-linear, first-order approximations of the expected value of the output random variable may not be reliable. Second, the quality of approximation

is hindered in case of large uncertainties in input variables, which is usually the case in quantitative risk estimation for geohazards.

4.2.4   First- and second-order reliability methods (FORM and SORM)

Hasofer & Lind (1974) proposed the first-order reliability method (FORM), which provides an invariant definition for the reliability index. The starting point for FORM is the definition of the performance function G(X), where X is the vector of basic random variables. If the joint probability density function of all random variables $F_x(X)$ is known, then the probability of failure P_f is given by

$$P_f = \int_L F_x(X)\, dX \tag{4.6}$$

where L is the domain of X where G(X) < 0.

In general, the above integral cannot be solved analytically. In the FORM approximation, the vector of random variables **X** is transformed to the standard normal space U, where U is a vector of independent Gaussian variables with zero mean and unit standard deviation, and where G(U) is a linear function. The probability of failure P_f is then (P[…] means probability that …):

$$P_f = P\,[G(U) < 0] \approx P\,[\sum_{i=1}^{n} \alpha_i U_i - \beta < 0] = \Phi\,(-\beta) \tag{4.7}$$

where α_i is the direction cosine of random variable U_i, β is the distance between the origin and the hyperplane G(U) = 0, n is the number of basic random variables X, and Φ is the standard normal distribution function.
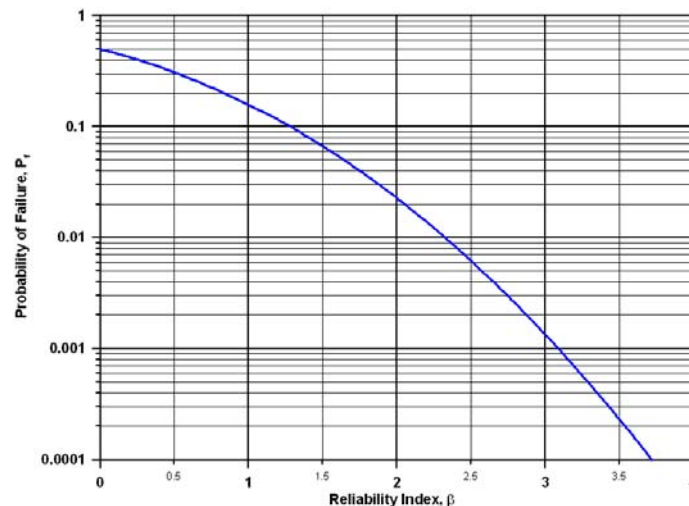
Figure 4.2 Relationship between reliability index β, and probability of failure P_f.

The vector of the direction cosines of the random variables (α_i) is called the vector of sensitivity factors, and the distance β is the reliability index. The probability of failure (P_f) can be estimated from the reliability index β using the established equation $P_f = 1 - \Phi(\beta) = \Phi(-\beta)$, where Φ is the cumulative distribution (CDF) of the standard normal variate. The

relationship is exact when the limit state surface is planar and the parameters follow normal distributions, and approximate otherwise. The relationship between the reliability index and probability of failure defined by Equation 4.7 is shown in Figure 4.2.

The square of the direction cosines or sensitivity factors (α_i^2), whose sum is equal to unity, quantifies in a relative manner the contribution of the uncertainty in each random variable X_i to the total uncertainty.

In summary the FORM approximation involves:
1. transforming a general random vector into a standard Gaussian vector,
2. locating the point of maximum probability density (most likely failure point, design point, or simply β-point) within the failure domain, and
3. estimating the probability of failure as $P_f \approx \Phi(-\beta)$, in which $\Phi(\cdot)$ is the standard Gaussian cumulative distribution function.

Example 4.2

Consider an infinite frictional soil slope with thickness H in the vertical direction, soil friction angle φ', slope angle θ, unit weight γ, and pore pressure u at depth H. With the following parameters and probability distribution functions, evaluate the probability of slope failure and its reliability index.

Parameter	μ	σ	Probability distribution
H (m)	10.0	1.0	Lognormal
φ' (degrees)	35.0	2.0	Normal
θ (degrees)	20.0	1.5	Lognormal
γ (kN/m³)	18.0	0.5	Normal
u (kPa)	20.0	7.0	Beta, between 0 and 40

The equation for the safety factor of the slope is:

$$F = \frac{\tan(\phi')}{\tan(\theta)} \cdot \left(1 - \frac{u}{\gamma \cdot H \cdot \cos^2(\theta)}\right)$$

A simple limit state function for the performance of the slope is:

$$G = F - 1$$

i.e. the slope is stable when the factor of safety is greater than 1, and it fails when the factor of safety is less than 1.

Using the software STRUREL (RCP, 1999), the following results are obtained:

Probability of failure: $P_f = 3.9 \cdot 10^{-5}$
FORM reliability index: $\beta = 3.95$

The sensitivity factors for the variables are shown on the figure below. The Latin symbols in table correspond to the following Greek symbols above: B is θ, phi is φ', and gam is γ. The pie chart shows the squares of the sensitivity factors α_i^2.

Representative Alphas of Variables FLIM(1), slope.pti

	u	-0.43
	B	-0.61
	phi	0.59
	gam	0.08
	H	0.30
	Sum of a²	1.00
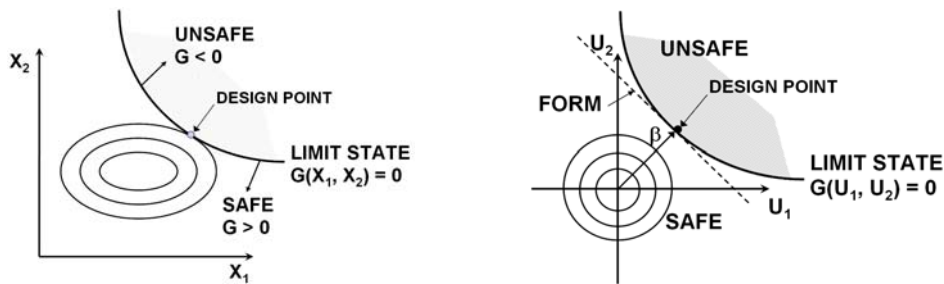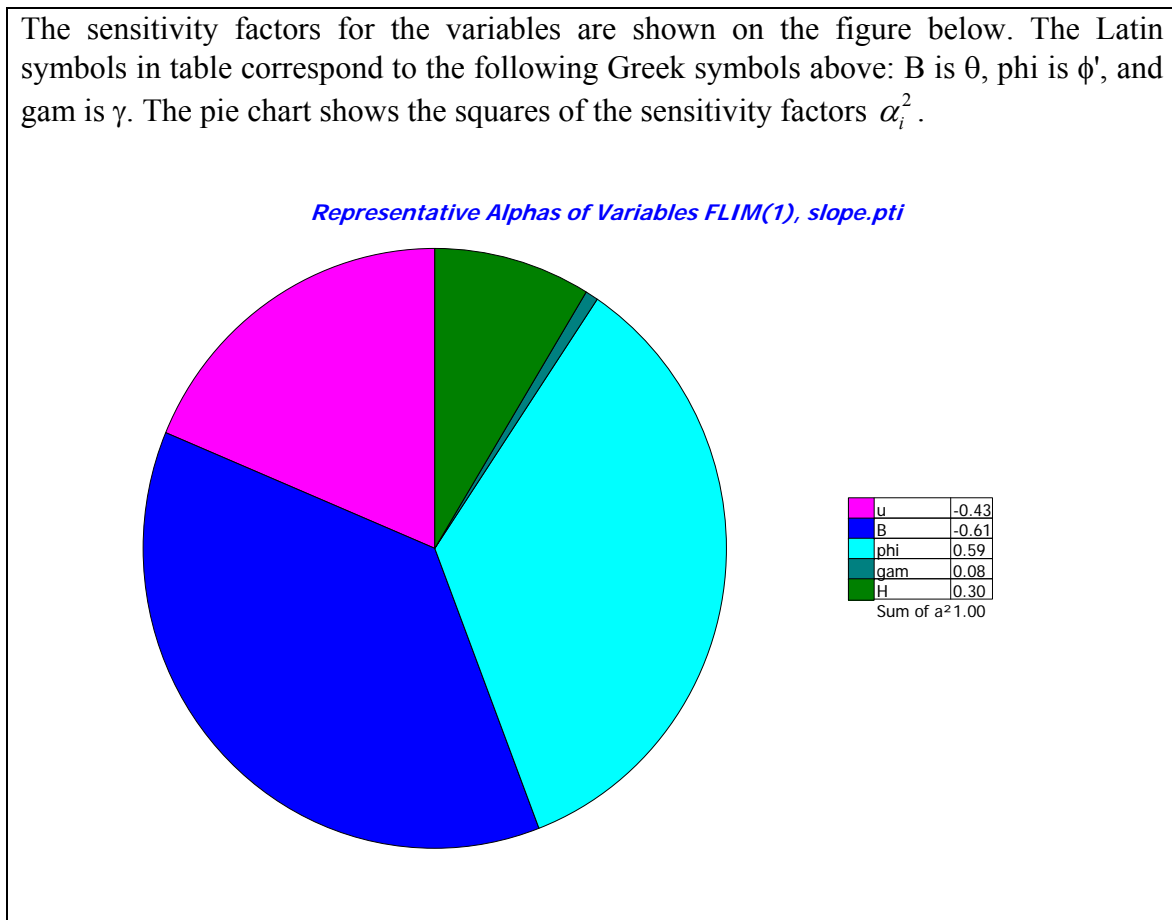
Figure 4.3 The FORM approximation (right) and definition of β and design point.

An illustration of the design point and graphical representation of β is given in Figure 4.3.

Low (2003) presented a method for finding the reliability index in the original space. His approach is based on the matrix formulation of the Hasofer-Lind reliability index β (Veneziano, 1974; Ditlevsen, 1981):

$$\beta = \min \sqrt{(X - \mu)^T C^{-1} (X - \mu)} \quad \text{for} \quad \{X : G(X) = 0\} \tag{4.8}$$

or, equivalently:

$$\beta = \min \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T [R]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]} \quad \text{for} \quad \{X : G(X) = 0\} \tag{4.9}$$

in which $X = (x_1, x_2, ..., x_n)$, μ = mean vector of X, C = covariance matrix of X, and R = correlation matrix of X.

Low and Tang (1997) used Equation 4.9 in preference to Equation 4.8 because the correlation matrix R is easier to set up, and conveys the correlation structure more explicitly than the covariance matrix C. Geometrically, for a two-variable problem, Equations 4.8 and 4.9 can be interpreted as finding the smallest ellipsoid (of the probability distribution of the variables) tangent to the limit state surface, see Figure 4.4. The key advantage of this formulation is that it can be implemented using built-in functions in EXCEL without programming and EXCEL is widely available on PCs (Phoon and Nadim, 2004).

In the second-order reliability method (SORM), the limit state function is defined as in FORM, but the resulting limit state function is approximated by a second order function (Breitung 1984). However, for geo-problems the probabilities of failure obtained with SORM analyses have been very close to the values obtained with FORM (Lacasse and Nadim, 1999).
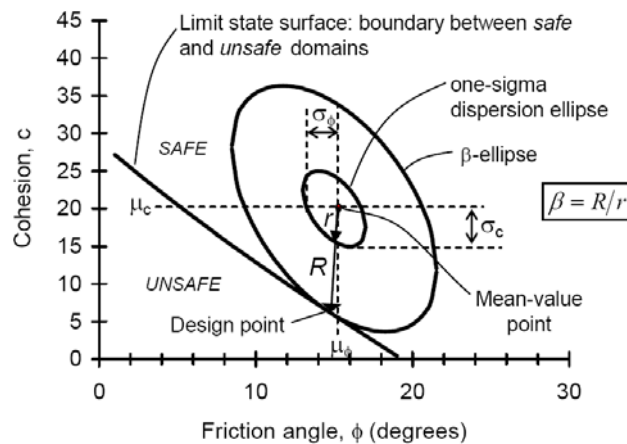
Figure 4.4 Illustration of β in the plane of original variables (Low, 2003).

4.2.5 System reliability

A system, for example a complex geotechnical structure, consists of many components or elements, whose individual or combined failure can lead to collapse. A simple gravity retaining wall would fail if the lateral forces on the wall exceed the base shear resistance (sliding mode of failure), if the weight of the wall and vertical forces acting on the wall exceed the bearing capacity at the base (bearing capacity mode of failure) or if the driving moment from the external loads exceeds the resisting moment from the weight of the wall (rotational mode of failure). The wall could therefore be thought of as a system that comprises three components whose individual failure would constitute the failure of the wall.

The methods and examples discussed in the previous section generally characterize the performance and reliability of a single component of a complete system. The failure event of a system, in a reliability sense, is defined as the union, or intersection, or combinations thereof, of component failure events. In a graphical representation of a system, the components are represented by boxes that are connected together by lines to form the system. Input and output are marked by arrows (see Figure 4.5).

(a) Series system   (b) Parallel system
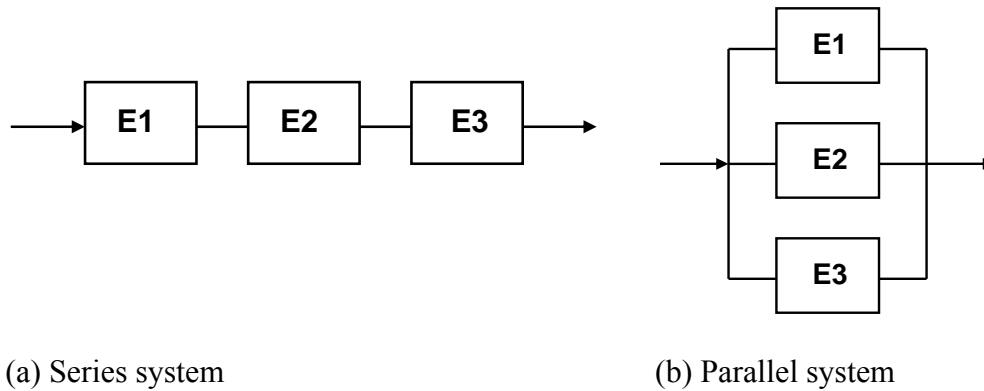
Figure 4.5 Schematic representation of series and parallel systems.

It is useful to distinguish between two basic types of systems depending on the logical structure of the components, namely series and parallel systems. In a series system the individual components are connected in series with regard to their function (Figure 4.5a). A series system will fail if any of its components fail, i.e. the system failure event is the union of all the component failure events. As a simple example, consider a chain consisting of many links. If the weakest link breaks, the chain fails. That is, the least reliable link determines the reliability of the system. If a series system is composed on "n" statistically independent components, then the probability of system failure can be computed from the probability of failure of individual components by the following equation:

$$P_{f,\text{system}} = 1 - \prod_{i=1}^{n}(1 - P_{f,i}) \quad \approx \quad \sum_{i=1}^{n} P_{f,i} \tag{4.10}$$

The summation approximation is valid for very small probabilities of failure $P_{f,i}$.

Obviously the probability of failure of a series system increases with the number of elements and is largely governed by the probability of failure of its most unreliable element. If all elements of a series system are perfectly correlated, then:

$$P_{f,\text{system}} = \max[P_{f,i}] \tag{4.11}$$

Thus the probability of failure of a series system lies within the following bounds:

$$\max[P_{f,i}] \ \leq \ P_{f,\text{system}} \leq 1 - \prod_{i=1}^{n}(1 - P_{f,i}) \tag{4.12}$$

In a parallel system, the elements of the system are connected in parallel with regard to their function (Figure 4.5b). This means that a parallel system will fail if all its components fail, i.e. the system failure event the intersection of the component failure events.

If a parallel system is composed on "n" statistically independent components, then the probability of system failure can be computed from the probability of failure of individual components by the following equation:

$$P_{f,system} = P_{f,1} \cdot P_{f,2} \cdot .... \cdot P_{f,n} = \prod_{i=1}^{n} P_{f,i} \tag{4.13}$$

If all elements of a parallel system are perfectly correlated, then:

$$P_{f,system} = \min[P_{f,i}] \tag{4.14}$$

That is, the probability of failure of a parallel system is never greater than probability of failure of its most unreliable element. The probability of failure of a parallel system, therefore, lies within the following bounds:

$$\prod_{i=1}^{n} P_{f,i} \leq P_{f,system} \leq \min[P_{f,i}] \tag{4.15}$$

In constructed facilities, true parallel systems are rare. Consider, for example, a foundation slab supported by six piles. This foundation system, on the first sight, might well be considered a parallel system consisting of six components, as all six piles must fail in order for the foundation to fail. However, the carrying capacities of the piles are strongly correlated. Furthermore, the question of ductile versus brittle failure of the components in the system is of prime importance. While a ductile component may continue to carry load until the other elements of the system yield, a brittle component stops carrying its share of load, leaving the remaining components with even more load.

Most real life systems are mixed systems that could be represented as a series connection of subsystems, where each subsystem comprises parallel components. Some commercial software for computation of system reliability (e.g. STRUREL) require that the system is represented is terms of minimal unions of intersections, also denoted as minimal cut-set.

4.3   HYBRID METHODS TO PROPAGATE RANDOMNESS AND IMPRECISION

In case one or many parameters are semi-quantitative and sufficient data are available to represent other input parameters of the model by probabilities, "hybrid" methods to jointly propagate randomness and imprecision can be used. For instance, the approach of Guyonnet et al. (2003) (and further developed by Baudrit et al. 2007) relies on the combination of Monte-Carlo techniques (random sampling of the probabilistic distribution assigned to random parameters) with Fuzzy interval analysis (applied to the possibility distribution assigned to the imprecise parameters, Dubois et al. 2000). The result of the propagation is summarized based on the general framework of the evidence theory (Shafer, 1976), hence producing a pair of probabilistic distributions (i.e. Plausibility and Belief distributions).

In the field of CO_2 storage risk assessment, Figure 4.6 depicts the pair of probabilistic indicators assigned to the CO_2 plume extension within the storage reservoir computed by

(Bellenfant et al. 2009) using the "hybrid" approach. Due to the imprecision of input parameters, the probability assigned to the CO_2 plume extension is not unique and lies within the interval defined by the Belief and the Plausibility indicators. For instance, the probability the CO_2 plume extension to be inferior to 13 km ranges from 70 to 100 % (area north). Note that if sufficient data were available, all input parameters would be represented by probabilistic distributions and the probability assigned to CO_2 plume extension would be unique.

The gap between both indicators represents a measure of the epistemic uncertainties (imprecision). Thus, this method provides guidelines for site characterization as it underlines regions were more data should be gained to reduce this gap, hence the imprecision.
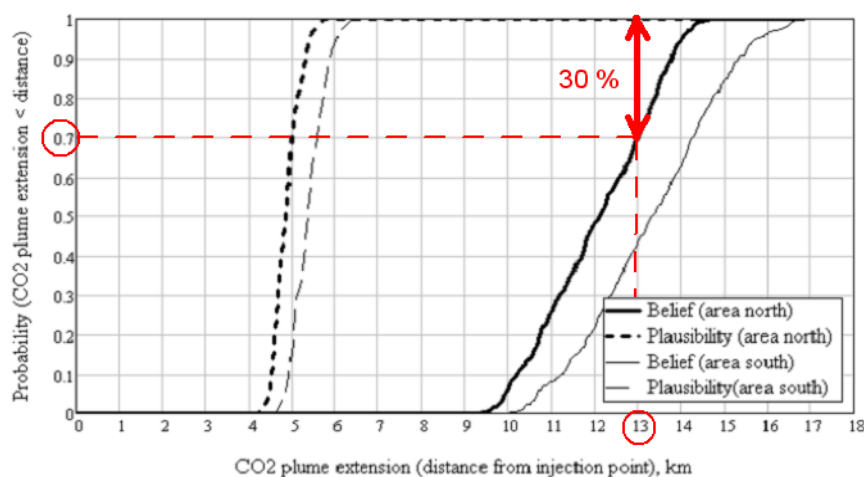
Figure 4.6 Pair of cumulative probabilistic distributions assigned to the CO_2 plume extension (Belief and Plausibility). The probability that the CO_2 extension is inferior to 13 km ranges from 70 and 100 %. Adapted from (Bellenfant et al. 2009)

4.4   EVENT TREE ANALYSIS

For a complex system, the conditions that could lead to any of the potential modes of failure may be quite involved and an event tree analysis is often the optimum way to quantify hazard and risk. Given a number of possible consequences resulting from an initiating event, the sequence of following events need to be identified and their probability of occurrence needs to be quantified. This can be done systematically and effectively through the use of an event tree diagram. The approach is widely used for dams, but is also useful for slopes with complex history, e.g. a rock slope with possibly different volumes sliding over time followed by a tsunami. Ang and Tang (1984) and Whitman (1984) presented several application examples for the method. Figure 4.7 illustrates event tree analysis.

A general event tree is shown in Figure 4.7 with an initiating event, E, and a number of possible consequences, $C_{ij...k}$. It can be observed that a particular consequence depends on the subsequent events following the initiating event; in other word, for a given consequence to occur, a sequence of subsequent events, or *path* in the event tree, must occur. Given an ini-

tiating event, there may be several "first subsequent events" that will follow. Obviously, these subsequent events are mutually exclusive. If we assume a particular first subsequent event, a mutually exclusive set of "second subsequent events" is possible. Each path in the event tree, therefore, represents a specific sequence of (subsequent) events, resulting in a particular consequence. The probability associated with the occurrence of a specific path is simply the product of the (conditional) probabilities of all the events on that path.
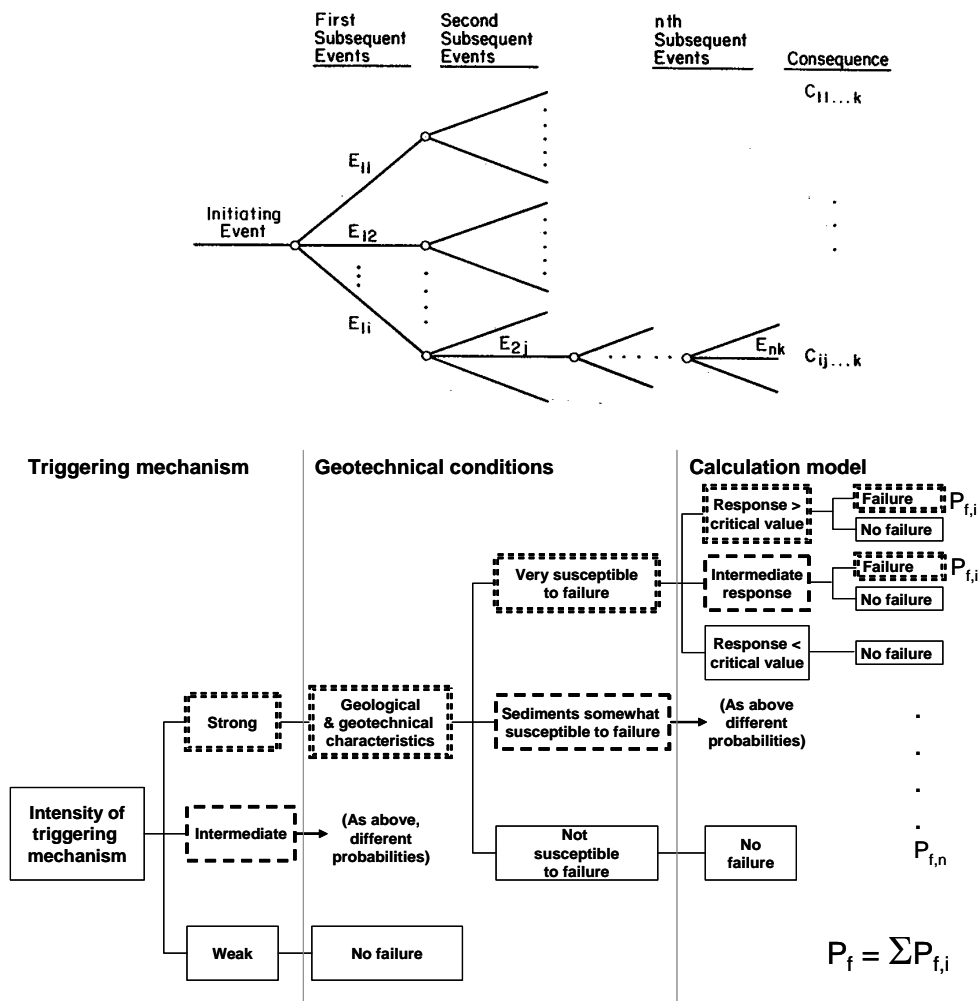
Figure 4.7 Event tree model and example for the analysis of a slope.

Each event in the event tree is associated with a probability of occurrence. The probabilities can be obtained by first assigning a verbal descriptor as given below. The sum of the probabilities at any node is always unity, if all possible events have been included. The estimates rely heavily on engineering judgment. Observations are also very useful in assisting judgment. Each outcome in the event tree ends up as failure or no failure. The total probability of failure is the summation of the probabilities of each outcome leading to failure. If data are available, component events should be treated statistically, for example the 100-year and 1000-year rainfall or flood, based on historic data, the earthquake frequency and response

spectrum. In practice, the results of an event tree analysis prove even more useful when done for several slopes and compared.

To achieve consistency in the evaluation of the probabilities from one dam to another, conventions have been established to anchor the probabilities. An example of descriptors of uncertainty used in the dam profession goes as follows:

Verbal description of uncertainty	Event probability
Virtually impossible	0.001
Very unlikely	0.01
Unlikely	0.10
Completely uncertain	0.50
Likely	0.90
Very likely	0.99
Virtually certain	0.999

Virtually impossible:	event due to known physical conditions or processes that can be described and specified with almost complete confidence.
Very unlikely:	the possibility cannot be ruled out on the basis of physical or other reasons.
Unlikely:	event is unlikely, but it could happen
Completely uncertain:	there is no reason to believe that one outcome is any more or less likely than the other to occur.
Likely:	event is likely, but it may not happen.
Very likely:	event that is not completely certain.
Virtually certain:	event due to known physical conditions or processes that can be described and specified with almost complete confidence.

The Intergovernmental Panel on Climate Change (IPCC, 2010) uses a similar scale for quantification of likelihood scales (Table 4.3). However, they recommend a probability range instead of a specific probability value. This conclusion is supported by several psychometric studies (see Piercey, 2009 and references therein). The issue of translating numerical probability values into words (and vice versa) is still an active research subject in the field of "Linguistic Probabilities" (e.g., Halliwell and Shen, 2009) relying on approaches of Fuzzy Theory.

Table 4.3 Calibrated language for describing quantified uncertainty (IPCC, 2010). Likelihood may be based on statistical or modelling analyses, elicitation of expert views, or other quantitative analyses.

Term	Likelihood of the Outcome
Virtually certain	99 - 100% probability
Very likely	90 - 100% probability
Likely	66 - 100% probability
About as likely as not	33 - 66% probability
Unlikely	0 - 33% probability

| Very unlikely | 0 - 10% probability |
| Exceptionally unlikely | 0 - 1% probability |

4.5 SUMMARY, PROBLEMS AND GAPS

Below, the MOVE (2011) framework in Figure 2.1 is used to categorize relevant methods for the treatment of uncertainties. Table 4.4 summarises the methodological information presented above.

Table 4.4 Summary of applicable approaches depending on "type of input data" and "type of method for the propagation of uncertainty". Recommended approaches for the different combinations of input data and methods are underlined (adapted from MOVE, 2011).

| | | Column | Type of Input data | | |
			A	B	C
	Row		*Qualitative*	*Semi-quantitative*	*Quantitative*
Method	1	*Implicit*	Expert judgment Qualitative scenario analysis	Multi-criteria decision analysis (including Fuzzy logic Bayesian theory/networks) NUSAP (Numerical, Unit, Spread, Assessment and Pedigree)	Fuzzy logic Bayesian theory/networks
	2	*Explicit*	Appropriate input data ranking procedure Monte-Carlo simulation Fuzzy logic NUSAP (Numerical, Unit, Spread, Assessment and Pedigree) Expert judgement	Monte-Carlo simulation Fuzzy logic NUSAP (Numerical, Unit, Spread, Assessment and Pedigree) Expert judgement Possibility theory and hybrid methods	Monte-Carlo simulation First-order second moment First-order reliability method NUSAP (Numerical, Unit, Spread, Assessment and Pedigree) Expert judgement

Reasoning for recommended approaches for different combinations of "input data" and "method" as summarised in Table 4.4:

Implicit methods: Cells A1, B1 and C1 in Table 4.4
- The Bayesian approach is a powerful tool, because it combines observations with expert judgement. Bayesian theory enables update and revise of belief values (values obtained from expert judgment) when new information (evidence) becomes available.
- Fuzzy logic is a very useful and flexible approach when data are missing, when very uncertain or only qualitative data is available and for very complex processes. It can

handle approximations and semantic notions with a logic based mathematical formulation.

- Expert judgment as an implicit method encompass unstructured and undocumented knowledge of experts

Explicit methods – qualitative input data: Cell A2 in Table 4.4

- To be able to use an explicit method to process qualitative input data, a ranking procedure must be applied to the input data, effectively transforming the input to a semi-quantitative or quantitative format. After ranking, the reader is referred to methods for semi-quantitative or quantitative data, see cells B2 or C2 respectively.

Explicit methods – semi-quantitative input data: Cell B2 in Table 4.4

- Fuzzy classification enables indicators to have a degree of membership in adjacent fuzzy sets, or in other words within two indicator categories. The propagation of uncertainty associated with input parameters is processed using fuzzy rules and the final categorisation is effectively performed by defuzzification.
- Monte Carlo with categorical data could be performed in two ways:
  a) Combine Monte Carlo Simulation and fuzzy logic, i.e. fuzzy classification. (This procedure is also classified as a hybrid method)
  b) If the categorisation is performed using score values in input and output parameters, Monte Carlo Simulation could be performed on the continuous score values in the input parameters. Then the categorisation is done on basis of the distribution of the output parameter scores.

In conclusion, the described procedures give both a parameter categorisation and additional information on the uncertainty associated with the categorisation.

Explicit methods –quantitative input data: Cell C2 in Table 4.4
The choice of method for treatment of uncertainties using explicit methods with quantitative input parameters depend on the formulation of the explicit method.

- Monte Carlo Simulation is the most flexible tool which can be used for all types of methods. It is available in several softwares and has the ability to provide the most accurate probability distribution of the output parameters. However, the method requires detailed data and a large number of simulations need to be run to obtain good results.
- If the parameter is formulated as a limit state function, the most convenient and reasonably accurate method for assessment of the uncertainties is the First Order Reliability Method (FORM).
- For other model formulations the First Order Second Moment (FOSM) method could be used as an approximation. FOSM is a simple and convenient method, for which only mean and standard deviation of the input parameters are needed. However, because the method is independent of the probability distributions of the input parameters, it provides approximations for the mean and standard deviation only and might be too simplistic when uncertainties in input parameters are large.

A general problem related to risk estimation is lack of data. Despite large uncertainties associated with natural processes that control hazard level, more uncertainty is often related to vulnerability than hazard when carrying out landslide risk assessment.

5   DEALING WITH UNCERTAINTY IN DECISION MAKING

5.1   INTRODUCTION

Decision making may be defined as the process of select making a logical choice from among several available options. Typical decision problems are subject to a combination of inherent, modelling and statistical uncertainties. When trying to make an appropriate good decision, a decision maker must weight the positives and negatives of each option, and consider all the alternatives. For effective decision making, a decision maker must be able to forecast the outcome of each option as well, and based on all these items, determine which option is the best for that particular situation. Most of decision theory is normative or prescriptive, i.e., it is concerned with identifying the best decision to take, assuming an ideal decision maker who is fully informed, able to compute with perfect accuracy, and fully rational.

The practical application of this prescriptive approach (how people actually make decisions) is called decision analysis. The objective of a decision analysis (DEA) is to discover the most advantageous alternative under the circumstances. Among management tools for decision analysis we find statistical tools such as decision tree analysis, multivariate analysis, and probabilistic forecasting. The most systematic and comprehensive software tools developed in this way are called decision support systems.

5.2   DECISION ANALYSIS (DEA)

Decision analysis using weight of evidence (WOE) can be defined as a framework for synthesizing individual lines of evidence, using methods that are either qualitative (examining distinguishing attributes) or quantitative (measuring aspects in terms of magnitude) to develop conclusions regarding questions concerned with the degree of impairment or risk. Even though all WOE methods may include qualitative and quantitative considerations, i.e implicit and explicit procedures, Linkov (2009) order the methods are by increasing degree of quantification (Table 5.1).

Table 5.1 Weight of evidence methods (Linkov 2009)

Method	Method description
Listing Evidence	Presentation of individual lines of evidence without attempt at integration
Best Professional Judgment	Qualitative integration of multiple lines of evidence
Causal Criteria	A criteria-based methodology for determining cause and effect relationships
Logic	Standardized evaluation of individual lines of evidence based on qualitative logic models
Scoring	Quantitative integration of multiple lines of evidence using simple weighting or ranking
Indexing	Integration of lines of evidence into a single measure based on empirical models
Quantification	Integrated assessment using formal decision analysis and statistical methods

Generally, Quantitative WOE methods statistically derive risk probabilities from several lines of evidence and use formal decision-analytical tools. When more than one method is depicted at the same level (e.g., Logic/Causal Criteria and Indexing/Scoring), this reflects that several methods have comparable quantitative rigor within the qualitative/quantitative continuum. A key consideration, however, is that neither Scoring nor Indexing quantifies judgments using formal decision analysis or probabilistic techniques. As a result, the transparency and reproducibility of these methods—as well as their ability to handle nonlinearity and correlation across criteria—serve as the delineating factors between Scoring/ Indexing and Quantitative methods. Quantification methods, unlike Scoring/Indexing methods, are able to integrate nonlinearity and correlations into their methodologies. They also allow transparent and reproducible integration of scientific results with individual expert or decision maker judgment (Section 4.1.1) and comparison across multiple experts. In cases with conflicting expert views, different strategies can be applied including making reliability estimates for each expert. Multi criteria decision analysis (MCDA) is an example of a quantification method that uses likelihoods to synthesize weights of evidence.

5.3   UNCERTAINTY IN DECISION ANALYSIS – AN ENGINEERING INTERPRETATION

A significant issue to deal with in decision analysis is to provide a clear and rational basis with regard to the cost-efficient safeguarding of personnel, environment, and assets in situations where uncertainties are at hand.

A classical example is the decision problem of choosing the height of a dike. The risk of dike flooding can be reduced by increasing the height of the dike; however, due to the inherent natural variability in the water level, a certain probability of dike flooding in a given reference period will always remain. Risk assessment within the theoretical framework of decision analysis can help in deciding on the optimal dike height by weighing the benefits of reduced dike flooding risks with the costs of increasing the dike height. However, a prerequisite for the risk assessment is that the means for assessing the probability of dike flooding are established, and this in turn requires that a probabilistic model for the future water level is available.

For the purpose of discussing the phenomenon of uncertainty in more detail, it is initially assume that the universe is deterministic and that our knowledge about the universe is perfect. This implies that it is possible by means of, e.g., a set of exact equation systems and known boundary conditions by means of analysis to achieve perfect knowledge about any state, quantity, or characteristic that otherwise cannot be directly observed or has yet not taken place. In principle, following this line of reasoning the future as well as the past would be known or assessable with certainty. Considering the dike flooding problem, it would thus be possible to assess the exact number of floods that would occur in a given reference period (the frequency of floods) for a given dike height and an optimal decision can be achieved by a cost benefit analysis.

Whether the universe is deterministic or not is a rather deep philosophical question. Despite the obviously challenging aspects of this question its answer is, however, not a prerequisite

for purposes of decision making, the simple reason being that even if the universe would be deterministic our knowledge about it is still in part highly incomplete and/or uncertain.

In decision analysis procedures subject to uncertainties such as Quantitative Risk Analysis (QRA) and Structural Reliability Analysis (SRA), a commonly accepted view angle is that uncertainties should be interpreted and differentiated in regard to their type and origin. In this way it has become standard to differentiate between uncertainties due to inherent natural variability, model uncertainties, and statistical uncertainties. As briefly discussed in Chapter 2, the first mentioned type of uncertainty is often denoted aleatory or Type 1 uncertainty and the two latter types are referred to as epistemic or Type 2 uncertainties. Without further discussion here it is just stated that, in principle, all prevailing types of uncertainties can be taken into account in engineering decision analysis within the framework of Bayesian probability theory; a more detailed treatment of this issue is given in Paté-Cornell (1996) and Lindley (1976). As discussed in Paté-Cornell (1996), while the challenge of dealing with aleatory uncertainties can also be addressed by e.g. frequentistic methods of probabilistic analysis, the use of a Bayesian approach provides a sound and holistic basis for handling both the aleatory and more importantly and crucially the epistemic uncertainties that stem from incomplete or imperfect knowledge about processes and phenomena in the universe.

Within this framework, it is useful to distinguish two fundamentally different types of uncertainties, namely epistemic and aleatory uncertainties. This distinction has been considered for risk assessment of technical systems, e.g., Apostolakis (1990) or Helton and Burmaster (1996), and increasingly for natural hazards, e.g., Hall (2003), Apel et al. (2004) or Straub and Der Kiureghian (2008), but has been discussed also for general geological applications by Mann (1993). Aleatory uncertainties are interpreted as random uncertainties, which, for a given model, are naturally inherent to the considered process; epistemic uncertainties are related to our incomplete knowledge of the process, often because of limited data and can be characterised in the form of model uncertainties and statistical uncertainties.

The absolute and relative magnitudes of aleatory and epistemic uncertainty are markedly case-specific. This differentiation in uncertainties is introduced for the purpose of setting focus on how uncertainty may be reduced, rather than calling for a differentiated treatment in the risk assessment and decision analysis process. The distinction is relevant because aleatory uncertainty cannot be reduced for a given model. In contrast, epistemic uncertainty can be reduced, for instance, by collecting additional information. For this reason, a clear identification of the epistemic uncertainties in the analysis is crucial, as these may be reduced at a later time. Furthermore, neglecting epistemic uncertainty can lead to strong underestimation of the risk, see Coles et al. (2003) for an example.

Considering again the dike example, an engineering model can be formulated where future extreme water levels are predicted in terms of a regression of previously observed annual extremes. In this case, the uncertainty due to inherent natural variability would be the uncertainty associated with the annual extreme water level. The model chosen for the annual extreme water level events would by itself introduce model uncertainties and the parameters of the model would introduce statistical uncertainties, as their estimation would be based on a limited number of observed annual extremes. Finally, the extrapolation of the annual extreme model to extremes over longer periods of time would introduce additional model

uncertainties. The uncertainty associated with the future extreme water level is thus composed as illustrated in Figure 5.1.

Whereas the so-called inherent natural variability is often understood as the uncertainty caused by the fact that the universe is not deterministic, it may also be interpreted simply as the uncertainty that cannot be reduced by means of the collection of additional information; see, e.g., Ditlevsen and Madsen (1996). It is seen that this definition implies that the amount of uncertainty due to inherent natural variability depends on the models applied in the formulation of the engineering problem. Presuming that a refinement of models corresponds to looking with more detail at the problem at hand, one could say that the uncertainty structure influencing a problem is scale dependent.

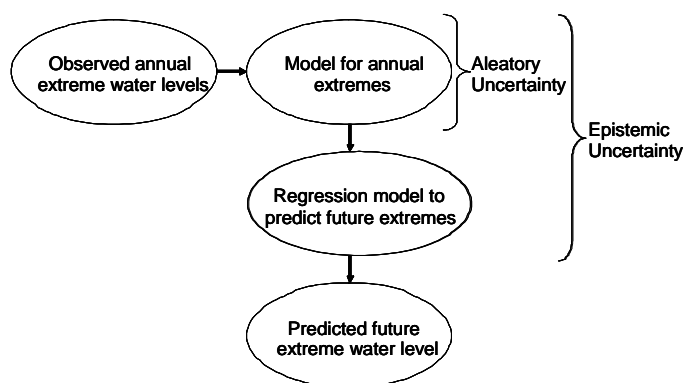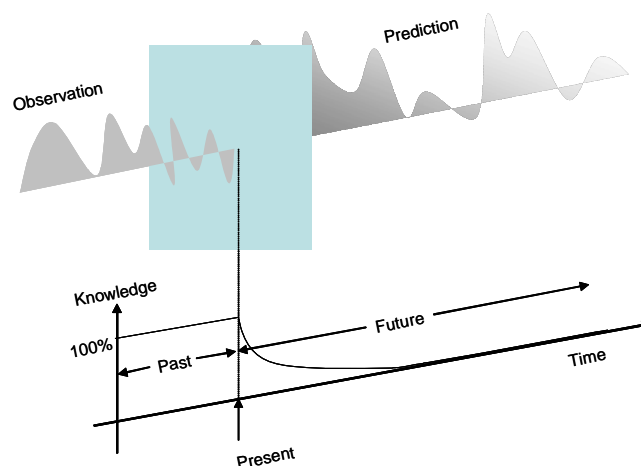*Figure 5.1 Illustration of uncertainty composition in a typical engineering problem*

Figure 5.2 Illustration of the time dependence of knowledge

Having formulated a model for the prediction of future extreme water levels and taking into account the various prevailing types of uncertainties, the probability of flooding within a given reference period can be assessed and just as in the case of a deterministic and perfectly

known universe we can decide on the optimum dike height based on a cost benefit assessment.

It is interesting to notice that the type of uncertainty associated with the state of knowledge has a time dependency. Following Figure 5.2, it is possible to observe an uncertain phenomenon when it has occurred. In principle, if the observation is perfect without any errors the knowledge about the phenomenon is perfect. The modelling of the same phenomenon in the future, however, is uncertain as this involves models subject to natural variability, model uncertainty, and statistical uncertainty. Often but not always the models available tend to lose their precision rather fast so that phenomena lying just a few days or weeks ahead can be predicted only with significant uncertainty. An extreme example of this concerns the prediction of the weather.

The above discussion shows another interesting effect, namely that the uncertainty associated with a model concerning the future transforms from a mixture of aleatory and epistemic uncertainty to a purely epistemic uncertainty when the modeled phenomenon is observed. This transition of the type of uncertainty has a significant importance because it facilitates the fact that the uncertainty is reduced by utilization of observation – referred to as updating.

5.4 BAYESIAN DECISION ANALYSIS AND PROBABILISTIC MODELLING

Consistent decision making subject to uncertainties is treated in detail in Raiffa and Schlaifer (1961) and Benjamin and Cornell (1970). In this section, an introduction to three different decision analyses is given – namely prior, posterior and pre-posterior decision analysis. Other aspects on decision analysis in engineering applications are treated in Apostolakis (1990), Paté-Cornell (1996) and Faber and Stewart (2003). An example on the modelling and management of uncertainties associated with rockfall hazards illustrating the concepts of Bayesian probabilistic modelling is provided in Chapter 6 of this report.

5.4.1 Prior decision analysis

The simplest form of the decision analysis is prior analysis. In the prior analysis, the risk (expected utility) is evaluated on the basis of statistical information and probabilistic modelling available prior to any decision and/or activity. This prior decision analysis is illustrated by a simple decision tree in Figure 5.3. In prior decision analysis the risk (expected utility) for each possible decision activity/option is evaluated in the principal form as:

$$R = E[U] = \sum_{i=1}^{n} P_i \, C_i \tag{5.1}$$

where R is the risk, U the utility, P_i is the ith branching probability (the probability of state i) and C_i the consequence of the event of branch i.

Prior decision analysis in fact corresponds closely to the assessment of the risk associated with an activity. Prior decision analysis thus forms the basis for the simple comparison of

risks associated with different activities. The result of a prior decision analysis might be that the risks are not acceptable and the risk reducing measures need to be considered.

In structural engineering a typical prior decision analysis is the design problem. A design has to be identified which complies with given requirements to the structural reliability. The representation of uncertainties is made on the basis of the existing information about materials and loads, however, as these have not occurred yet the probabilistic modelling involve both aleatory and epistemic uncertainties. As a general comment it should be noted that in the context of setting requirements to reliability and risk it is necessary to ensure consistency between the probabilistic models used for setting the requirements and the probabilistic models used for their verification.
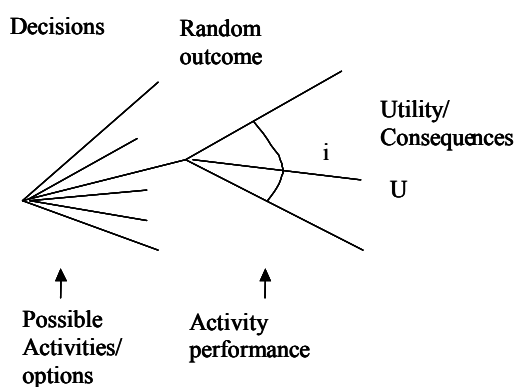
Figure 5.3 Decision tree for prior and posterior decision analysis

5.4.2   Posterior decision analysis

Posterior decision analysis is in principle of the same form as the prior decision analysis, however, changes in the branching probabilities and/or the consequences in the decision tree reflect that the considered problem has been changed as an effect of e.g. risk reducing measures, risk mitigating measures and/or collection of additional information. Posterior decision analysis may thus be used to evaluate the efficiency of risk reducing activities with known performances. The posterior decision analysis is maybe the most important in engineering applications as it provides a means for the utilization of new information in the decision analysis – referred to as updating; this is described in short in the following.

5.4.2.1   *Uncertainty updating – updating of random variables*

Inspection or test results relating directly to realizations of random variables may be used in the updating. The distribution parameters are initially (and prior to any update) modeled by prior distribution functions.

By application of Bayes theorem, see e.g. Lindley (1976), the prior distribution functions, assessed by any mixture of frequentistic and subjective information, are updated and transformed into posterior distribution functions.

Assume that a random variable X has the probability distribution function $F_X(x)$ and density function $f_X(x)$. Furthermore assume that one or more of the distribution parameters, e.g. the mean value and standard deviation of X are uncertain themselves with probability density function $f_Q(q)$. Then the probability distribution function for Q may be updated on the basis of observations of X, i.e. $\hat{x}$.

The general scheme for the updating is:

$$f_Q''(q \mid \hat{x}) = \frac{f_Q'(q) L(q \mid \hat{x})}{\int\limits_{-\infty}^{\infty} f_Q'(q) L(q \mid \hat{x}) \, dq} \qquad (5.2)$$

where $f_Q(q)$ is the distribution function for the uncertain parameters Q and $L(q|\hat{x})$ is the likelihood of the observations or the test results contained in $\hat{x}$. Here $''$ denotes the posterior and $'$ the prior probability density functions of Q. The observations $\hat{x}$ may not only be used to update the distribution of the uncertain parameters Q, but also to update the probability distribution of X. The updated probability distribution function for X $f_X^U(x)$ is often called the predictive distribution or the Bayes distribution. The predictive distribution may be assessed through

$$f_X^U(x) = \int\limits_{-\infty}^{\infty} f_X(x|q) f_Q''(q \mid \hat{x}) \, dq \qquad (5.3)$$

In Raiffa and Schlaifer (1961) and Aitchison and Dunsmore (1975) a number of closed form solutions to the posterior and the predictive distributions can be found (also collected in JCSS (2000)) for special types of probability distribution functions known as the natural conjugate distributions. These solutions are useful in updating of random variables and cover a number of distribution types of importance for reliability based structural reassessment. However, in practical situations there will always be cases where no analytical solution is available. In these cases FORM/SORM techniques as discussed in Section 4.2.4 (also in Madsen et al. (1986)) may be used to integrate over the possible outcomes of the uncertain distribution parameters and in this way allow for assessing the predictive distribution.

5.4.2.2 Probability updating - updating of uncertain relations

In many practical problems the observations made of realizations of uncertain phenomena cannot be directly related to random variables. In such cases other approaches must be followed to utilize the available information. Given an inspection result or other observation of an outcome of a functional relationship between several basic variables, probabilities may be updated using the definition of conditional probability or its extension known as Bayes formula:

$$P(F|I) = \frac{P(F \cap I)}{P(I)} = \frac{P(I|F)P(F)}{P(I)} \qquad (5.4)$$

F = Failure
I = Inspection result

For a further evaluation of Equation (5.4) it is important to distinguish between the types of inspection results. For inequality type information which may be expressed by limit states of the following form $h(\mathbf{X}) < 0$, Equation (5.4) may be elaborated in a straightforward way. Let *F* be represented by $M(x) \leq 0$, where *M* denotes the safety margin. We then have:

$$P(F|I) = \frac{P(M(\mathbf{X}) \leq 0 \cap h(\mathbf{X}) < 0)}{P(h(\mathbf{X}) < 0)} \qquad (5.5)$$

where X = vector of random variables having the prior distribution $f_{\mathbf{X}}(\mathbf{x})$. This procedure can easily be extended to complex failure modes and to a set of inspection results ($\cap h_i(x) < 0$).

5.4.3   Pre-posterior decision analysis

Using pre-posterior decision analysis optimal decisions in regard to information collection activities, which may be performed in the future, can be identified. Pre-posterior decision analysis is excellently described in e.g. Raiffa and Schlaifer (1961) and Benjamin and Cornell (1970). The principle behind the pre-posterior decision analysis is that the outcomes of planned information collection activities are assumed to follow the prior probabilistic model of uncertainties. Based on these assumed outcomes and taking into account any uncertainties associated with the observation and/or interpretation of the outcomes posterior decision analyses are performed. The corresponding risks are thereafter weighed with their probability of occurrence, again based on the prior probabilistic modelling. The pre-posterior may thus be interpreted as a posterior decision analysis made before new information is actually collected. The principle is also illustrated by the decision tree shown in Figure 5.4. An important pre-requisite for pre-posterior decision analysis is that decision rules specifying future actions which will be taken on the basis of the results of the planned information collection activities need to be formulated.
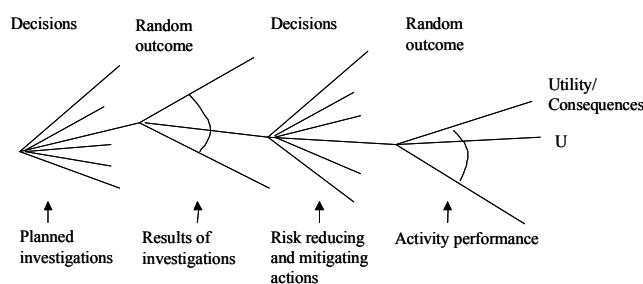

Figure 5.4 Decision tree for pre-posterior decision analysis

In pre-posterior decision analysis, the risk (expected utility) for each of the possible risk reducing activities is evaluated as

$$
\begin{aligned}
R &= E[U] \\
&= \min_{a} E'_{\mathbf{Z}} \left[E''_{|\mathbf{Z}} \left[U(a(\mathbf{z}), \mathbf{z}) \right] \right] \\
&= \min_{a} E'_{\mathbf{Z}} \left[\sum_{i=1}^{n} P_i''(a(\mathbf{z}), \mathbf{z}) C_i(a(\mathbf{z})) \right]
\end{aligned}
\tag{5.6}
$$

where $a(\mathbf{z})$ are the decision rules describing the different possible actions that can be take on the basis of the result of the considered investigation $\mathbf{z}$, $E[\ ]$ is the expected value operator. $P_i(\ )C_i(\ )$ is the product of the probability of the i'th event resulting from the decision and the corresponding consequences. $'$ and $''$ refer to the probabilistic description of the events of relevance based on prior and posterior information respectively, see e.g. Lindley (1976). Pre-posterior decision analysis forms a strong decision support tool and has been intensively used for the purpose of risk based inspection planning see e.g. Faber (2002). However, so far pre-posterior decision analysis has been grossly overlooked in risk assessments in general.

5.4.4   Uncertainty representation in updating

As mentioned earlier, it is important to differentiate between the different types of uncertainty in the probabilistic modelling of uncertain phenomena. Only when the origin and the nature of the prevailing uncertainties are fully understood a consistent probabilistic modelling can be established allowing for rational decision making regarding risk reduction by means of posterior and pre-posterior decision analysis. In the following the representation of uncertainties for representative posterior and pre-posterior decision problems is thus addressed and discussed.

5.4.4.1   Uncertainty modelling in posterior decision problems

In engineering decision analysis posterior decision problems typically involve the updating of the probability of a future adverse event F, P_F^U conditional on the observation of an event I which can be related to the adverse event. Such observations may in general be considered as being indications about the adverse event. The probability P_F^U may be assessed by Equation (5.4):

$$
P_F^U = P(F|I) = \frac{P(F \cap I)}{P(I)}
\tag{5.7}
$$

Taking basis in Equation (5.7) a simple case is now considered where the adverse event is a future ($\tau \in [t, T]$) failure event in terms of a load $S(\tau)$ exceeding the resistance R of an existing structural component. Furthermore it is assumed that the indicator I is the event that the component has survived all previous realizations of the loading $S(\tau)$ $\tau \in [0, t]$. Then Equation (5.7) can be written as:

$$P_f^U(T) = P(F|I) =$$

$$\frac{P\left(\left\{\min_{\tau \in [t,T]}(R - S(\tau)) \leq 0\right\} \cap \left\{\min_{\tau \in [0,t]}(R - S(\tau)) > 0\right\}\right)}{P\left(\left\{\min_{\tau \in [0,t]}(R - S(\tau)) > 0\right\}\right)} \qquad (5.8)$$

In accordance with the considerations made in the previous R is an epistemic uncertainty since it has already had its realization but it is still unknown and thus uncertain. As long as no other information is available it would be consistent to model the epistemic uncertainty associated with R using the same model assumptions (distribution type etc.) as before R had its realization. $S(\tau)$ is "in principle" an aleatory uncertainty (assuming that no model and/or statistical uncertainties are involved in the modelling of the load) when we consider future loads i.e. for $\tau \in [t,T]$. The uncertainty associated with $S(\tau)$ is of an epistemic nature when we consider already occurred load events, i.e. $\tau \in [0,t]$. The wording "in principle" is used because the temporal dependency characteristics of the loading $S(\tau)$ play a significant role. If the load events (or extreme loads) in consecutive time intervals are assumed to be conditional independent – a relatively normal case in engineering problems – then the consideration outlined in the above are valid. This also implies that the uncertainty associated with the future loading cannot be updated on the basis of observations of the past loading. However, if the load events in consecutive time intervals are dependent then a part of the uncertainty associated with the future loading becomes epistemic as soon as its first realization has occurred. The "size" of the part depends on the temporal dependency.
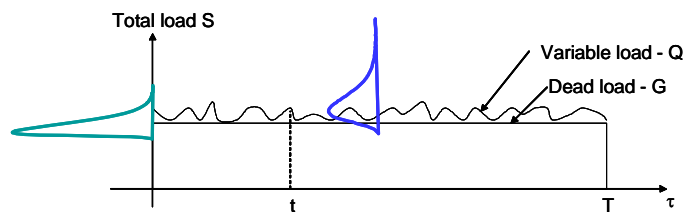
Figure 5.5 Illustration of a load with high degree of temporal dependency

In Figure 5.5, it is illustrated how the load events in consecutive time intervals may be highly dependent due to e.g. a dominating dead load component. Before the dead load component is realized the loading in the future might be subject to aleatory uncertainty only. As soon as the dead load component is realized a large part of the uncertainty associated with the future loading becomes epistemic. This effectively implies that this part of the uncertainty associated with the future loading can be updated on the basis of observations of the past loading. In other words – the part that can be updated is exactly the epistemic part of the uncertainty. If the probabilistic modelling of the uncertainties and the probability updating is performed in accordance with Equation (5.8) and the considerations outlined in the above then, the resulting probabilistic modelling and the representation of the different types of uncertainties is consistent. However, if in the representation of the adverse event and the updating event the different types of uncertainty and the temporal dependency is not consistently taken into account, the results may become grossly erroneous and non-physical.

5.4.4.2 *Uncertainty modelling in pre-posterior decision problems*

As can be realized from Equation (5.6) pre-posterior decision problems may be seen as a series of posterior decision problems for which the optimal solutions are averaged out over the entire prior uncertainty. The formulation of each of the posterior decision problems is based on an updated probabilistic model of the prevailing uncertainties assuming a given "outcome of nature". Therefore the considerations made for posterior decision analysis, concerning the treatment of uncertainties are also valid for pre-posterior decision problems.

5.4.5 Use of Bayesian probabilistic networks (BPN)

The risk assessment and management of natural hazards such as landslide and rockfall events requires a systematic and consistent representation and management of information for a typically complex system with a large number of constituents or sub-systems. Such representation must enable a rational treatment and quantification of the various uncertainties discussed earlier; these uncertainties can be associated with the constituents as well as the system. The consistent handling of new knowledge about the system and its constituents as and when it becomes available and its use in the risk assessment and decision making process is also essential. Further, the numerous dependencies and linkages that exist between different constituents of the system need to be systematically considered. The above requirements and considerations necessitate the use of generic risk models for the assessment and management of risks due to natural hazards. The use of Bayesian Probabilistic Networks (BPNs) has proven to be efficient in such risk assessment applications (Graf et al., 2009; Faber et al., 2007; Nishijima and Faber, 2007; Bayraktarli et al., 2006; Bayraktarli et al., 2005; Faber et al. 2005; Schubert et al, 2005 and Straub, 2005). A brief overview of the principles and use of Bayesian Probabilistic Networks is provided below; details can be found in Jensen (2001).

Formally, Bayesian probabilistic networks (BPN) are directed acyclic graphs whose nodes represent random variables in the Bayesian sense: they may be observable quantities, latent variables, unknown parameters or hypotheses. Edges represent conditional dependencies; nodes which are not connected represent variables which are conditionally independent of each other. Each node is associated with a probability function that takes as input a particular set of values for the node's parent variables and gives the probability of the variable represented by the node. Efficient algorithms exist that perform inference and learning in BPNs. Using a BPN offers many advantages over traditional methods of determining causal relationships. Independence among variables is easy to recognize and isolate while conditional relationships are clearly delimited by a directed graph edge: two variables are independent if all the paths between them are blocked (given the edges are directional).

5.5 OTHER QUANTITATIVE DEA METHODS

5.5.1 Event tree analysis

Event tree analysis is a widely used approach in quantitative DEA. This approach is described in Section 4.4.

5.5.2 Multi criteria decision analysis (MCDA)

The common purpose of MCDA methods is to evaluate and choose among alternatives based on multiple criteria using systematic analysis that overcomes the limitations of unstructured individual or group decision-making. According to Linkov et al (2007) MCDA may be divided in:

- Value function-based methods (MAVT/MAUT)
- Outranking methods.
- Analytical hierarchy process (AHP)

Value function-based method is to model and represent the decision maker's preferential system into a value function V(**a**),

$$V(\mathbf{a}) = F(V_1(a_1),\ldots,V_m(a_m)) \qquad (5.9)$$

where alternative a is presented as a vector of the evaluation criteria $a=(a_1,\ldots,a_m)$, a_i is the assessment of alternative **a** according to criterion i, and $V_i(a_i)$ is the value score of the alternative reflecting its performance on criterion i (as a rule, $0 \leq V_i(a_i) \leq 100$). The most widely used form of function F( ) is an additive model:

$$V(\mathbf{a}) = w_1 V_1(a_1) +\ldots+ w_m V_m(a_m) \qquad (5.10)$$

$$w_i > 0 \,, \sum w_i = 1 \qquad (5.11)$$

where w_i, i=1,…,n, are the weights reflecting the relative importance of criteria (or corresponding scaling factors. It should be stressed, however, that for a justified implementation of the additive model (Equation 5.10) some requirements/axioms of MAVT should be held (among them the key ones are the preferential independence requirements). MAVT relies on the assumption that the decision-maker is rational (preferring more utility to less utility, for example), that the decision-maker has perfect knowledge, and that the decision-maker is consistent in his judgments. The goal of decision-makers in this process is to maximize the overall value V(**a**) of alternative **a.** Uncertainties are considered in value based methods by incorporating a non linear value function instead of simple linear normalisation the preferences may be expressed with a higher degree of certainty than a linear approximation.

A description of the outranking methods can be found in Alvarez (2010). Outranking methods are partially compensatory and can better relate to the decision-maker needs, as they are adapted to preference structures in certain types of decisions. Since outranking is a partly compensatory method, uncertainties may be incorporated within the variation of parameters expressing the probability of achieving a certain preference order.

5.6   THE DECISION MAKING PROCESS

5.6.1   Weighing of criteria

In a formal decision analysis handling of uncertainties is of significant importance. Whereas uncertainties in the data material, methodologies and criteria may be handled with various analytical processes there is still a subjective part of weighing in decision analysis. This may be simple scoring performed by experts in a consensus setting or a formal multi-criteria involvement process as described in Sparrevik et al 2011 (Figure 5.6).
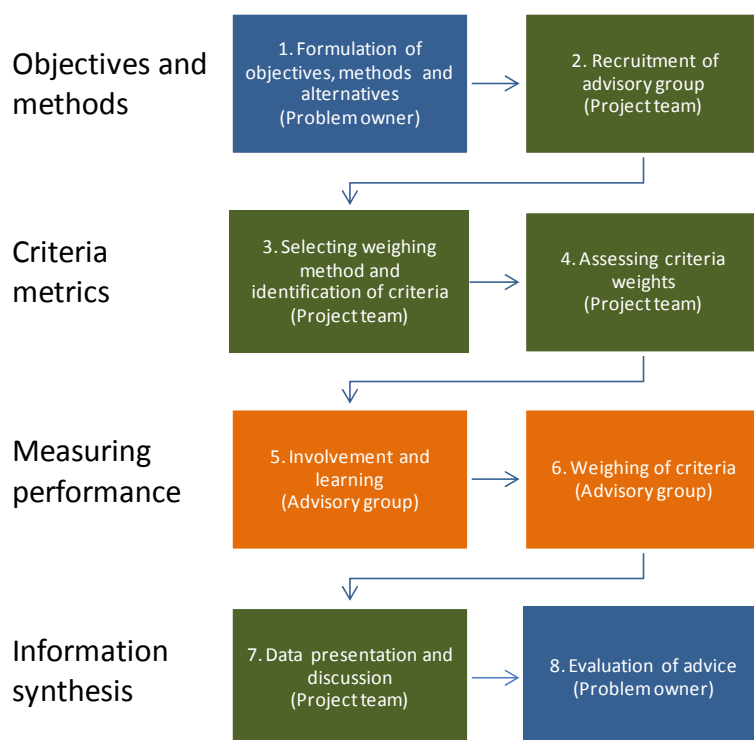
Figure 5.6 Using a multi criteria involvement process for weighing of criteria

5.6.2   Methods for weighing

At the heart of decision analysis is the idea that by quantifying the preferences about the goals rather than the solution, that a more objective, systematic solution can be reached. This quantification of the goals is done by the weighting of criteria; by setting which judging criteria are most important, we are constructing the framework by which the potential solutions will be measured. Several methods can be utilised for weighing:

- Ranking could be considered the basic weight elicitation scheme. In this case each criterion a percentage that indicates its importance; the sum of the percentages across all criteria must equal 100%.

- Pair-wise comparison is another method of weight elicitation which is specific to the analytic hierarchy process. In this method, each criterion is compared to another and the dominance is assessed. By using a mathematical algorithm the criteria are ranked in order. The idea with pair-wise comparison instead of direct ranking is the assumption that decision makers are more relaxed in relative comparison than ranking in absolute order. Since pair-wise weighing among criteria opens for inconsistency this has to be evaluated as a part of the uncertainty analysis by computing the inconsistency index. Perfect consistency will give an inconsistency index of 0, whereas high numbers indicate high degree of inconsistency. Often a threshold value of 0.1 is recommended

- Swing-weighting is an alternative to ranking, but produces similar results. In this method people are first asked to arrange the preferential order of criteria by elevating one criterion at a time from a bottom level to the highest level of preference. When the preference order between criteria is determined the relative preference of each criteria is assessed against the highest ranked criteria (given a value of 100) on a 1-100 scale.

6   EXAMPLE – MODELLING OF ROCKFALL HAZARDS BASED ON A BAYESIAN APPROACH

6.1   INTRODUCTION

An example taken from Straub and Schubert (2008) involving the modelling of rockfall hazards and design of rockfall protection structures is considered here.

Rockfall is generally considered an inherently uncertain process, i.e., it is not possible to deterministically predict the time and the extent of the next event. However, it is possible to describe rockfall using a probabilistic model, describing the frequency $H_V(v)$ with which a rock of a certain volume V or larger is detached. Because the assessment of rockfall is based on limited data and simplified models, the probabilistic model is itself subject to uncertainty itself; this can be represented by modelling the parameters of $H_V(v)$ as random variables. In this case, we write $H_V(v|\boldsymbol{\theta})$ to indicate that the model is defined conditional on the values of its parameters $\boldsymbol{\theta}$. This epistemic uncertainty on $\boldsymbol{\theta}$ can be depicted by credible intervals (which can be considered as the Bayesian equivalent of confidence intervals) on the exceedance frequency curve as demonstrated in Figure 6.1.
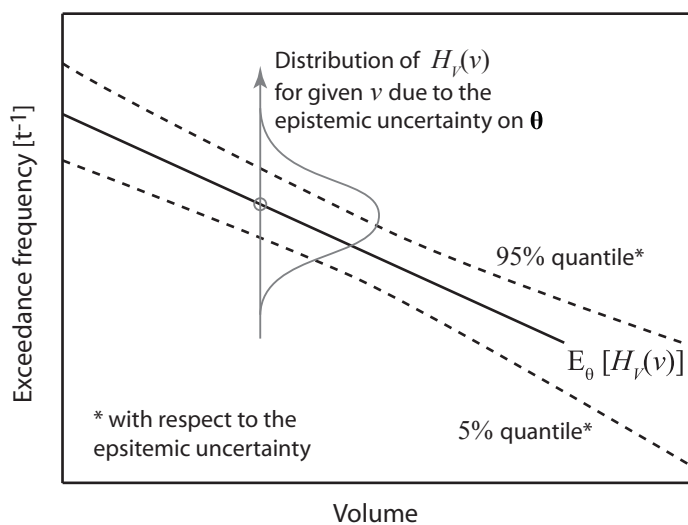
Figure 6.1 Exceedance frequency – illustrating the difference between epistemic and aleatory uncertainty

6.2   UNCERTAINTIES IN ROCKFALL HAZARDS

As with most natural hazards, the uncertainties related to the occurrence of the hazard are generally large for rockfall hazards. In the literature, this uncertainty is generally represented by an exceedance frequency as illustrated in Figure 6.1, yet without explicit consideration of the epistemic uncertainty. Instead it is (implicitly) assumed that the frequency of an event with a certain rock volume is a deterministic value, implying that if the site were observed over a sufficiently long period, exactly the predicted frequency of rocks would be

experienced. Clearly, this is not the case; instead the predicted frequency is a best estimate of the true rate of occurrence.

In the literature, various methods are proposed for identifying the exceedance frequency at a specific site. These include:

i)   the analysis of historical datasets, e.g., Hungr et al. (1999) or Dussauge-Peisser (2002),
ii) empirical models which describe hazard as a function of different indicators (observable parameters) such as topography and geology, e.g., Budetta (2004) or Baillifard et al. (2004),
iii) phenomenological (mechanical) models, e.g., Jimenez-Rodriguez et al. (2006) or Duzgun et al. (2003), and
iv) expert opinion, e.g., Schubert et al. (2005).

All these methods are useful in a particular context. While methods i) and ii) are generally more appropriate for the analysis of larger areas with less accuracy, iii) and iv) are more suited for the detailed analysis of a specific site.

Large-scale models (i) and ii) above) are generally based on statistical methods. Consequently, it is mathematically convenient to express the exceedance frequency in a parametric format. Traditionally, a power law has been applied to describe the relation between rock volume V and the exceedance frequency:

$$H_V\left(v|\mathbf{\theta}\right) = av^{-b}$$
(6.1)

The statistical parameters of the model characterising the shape of the exceedance frequency curve are $\mathbf{\theta} = [a,b]^{\mathrm{T}}$. The epistemic uncertainty is included in the analysis by modelling $\mathbf{\theta}$ as a random vector. Using the probability density function of $\mathbf{\theta}$, $f_{\mathbf{\Theta}}(\mathbf{\theta})$, the unconditional exceedance frequency is computed as:

$$H_V\left(v\right) = \int_{\mathbf{\Theta}} f_{\mathbf{\Theta}}\left(\mathbf{\theta}\right) H_V\left(v|\mathbf{\theta}\right) d\mathbf{\theta}$$
(6.2)

There are various sources for epistemic uncertainties in large scale models, preventing an exact prediction of the exceedance frequency for a particular site. A brief description of these is provided below.

Statistical uncertainty
The parameters of the large scale models are derived empirically from data sets. Because of the limited size of these data sets, the estimated parameters are subject to statistical uncertainty.

Measurement uncertainty
Measurements and recordings of the geological properties are typically subject to uncertainty and observations of historical events are often incomplete and biased and must rely on local experts. As an example, rocks on a road will generally be reported and documented, but those

that missed the road may often not be. Measurement uncertainty also results from derives from equipment, operator/procedural and random measurement effects.

Model uncertainty

Extrapolation of the statistical models to areas other than those for which observations are available leads to additional uncertainty as the geological and topographical characteristics will be different for these areas. GIS-based models will take into account some of these parameters, but the omitted parameters will lead to an uncertainty in the model predictions. Uncertainty also occurs due to the approximations and simplifications inherent in empirical, semi-empirical, experimental or theoretical models used to relate measured quantities to non-measurable numerical parameters used in estimation. Finally, although the power-law is, for example, commonly assumed to express exceedance frequency in the case of rockfall hazard, it has not been justified by phenomenological considerations. Thus, it is not ensured that the parametrical model accurately represents the actual behaviour.

Spatial variability

The frequency of hazard events varies in space. The observations represent an average over an area and the resulting parameter values, therefore, do not reflect the variations from the average.

Temporal variability

The frequency of hazard events varies in time. When working with annual frequencies, the seasonal changes do not affect the analysis, but the frequency may change over the years or may be dependent on extreme events (e.g., earthquakes). However, in certain instances, e.g., when temporal closure of the road is considered as a risk reduction measure, seasonal variations must be explicitly addressed by the analysis.

How can these uncertainties be quantified? Statistical uncertainty can be quantified by using standard statistical methods such as Bayesian analysis, see, e.g., Coles (2001). Measurement uncertainty can generally be estimated when the data collection method is known. Unfortunately, no simple analytical method is available for estimating model uncertainties. A solution is to rely on expert opinion, i.e., to ask experts about their confidence in the models. It is also possible to compare the model with observations which have not been used in the calibration of the model (model validation) or to compare different models. Furthermore, it is possible to include additional parameters in the formulation of the exceedance frequency. The model uncertainties are then reduced while the statistical uncertainties increase, but the latter can then be estimated analytically. Coles et al. (2003) demonstrate this for the analysis of rainfall data. The spatial and temporal variability can be analysed quantitatively, if data is available in sufficiently small scale; a data-set showing the spatial distribution of rockfall events is presented in Dussauge-Peisser et al. (2002). Spatial variability can be described by the spatial correlation of the relevant characteristics. In most practical cases, however, a simplified approach is favourable, whereby smaller areas are determined within which the spatial variability can be neglected. Temporal (typically seasonal) variability can be described by time-dependent parameters in the exceedance frequency model, corresponding to the assumption of the hazard event (e.g. rockfall) following an inhomogeneous Poisson process.

For small-scale models, the application of the power-law is not always justified, in particular if different mechanisms are underlying the detachment of smaller and larger rocks. In such cases it might be more appropriate to utilize a non-parametric model in which the rock

volume is divided into a discrete number of intervals (e.g., $10m^3 - 50m^3$) and the model gives the annual frequency of rocks for the different volume ranges.

6.3 BAYESIAN ANALYSIS AND UPDATING

For the modelling of rockfall exposure, Bayesian analysis is particularly useful, as it facilitates the consistent combination of different information in a single model. This is because the probabilistic model can be updated when new information becomes available. Consider the case where rockfall exposure at a particular location is expressed by the model $H_V(v|\boldsymbol{\theta})$ with uncertain parameters $\boldsymbol{\theta}$. When new information becomes available (denoted by $\mathbf{z}$), the probability distribution of the uncertain parameters can be updated using Bayes' theorem, which in its general form can be written as:

$$f_{\boldsymbol{\Theta}}\left(\boldsymbol{\theta}|\mathbf{z}\right) \propto L\left(\boldsymbol{\theta}|\mathbf{z}\right) f_{\boldsymbol{\Theta}}\left(\boldsymbol{\theta}\right) \tag{6.3}$$

$f_{\boldsymbol{\Theta}}\left(\boldsymbol{\theta}\right)$ is the prior probabilistic model, $f_{\boldsymbol{\Theta}}\left(\boldsymbol{\theta}|\mathbf{z}\right)$ is the updated model and $L\left(\boldsymbol{\theta}|\mathbf{z}\right)$ is the likelihood function, which describes the new information. The proportionality constant is obtained from the fact that integration of $f_{\boldsymbol{\Theta}}\left(\boldsymbol{\theta}|\mathbf{z}\right)$ over the entire domain of $\boldsymbol{\theta}$ must yield one. The likelihood function is the probability of the observed information given the parameters $\boldsymbol{\theta}$, i.e.,

$$\mathrm{L}\left(\boldsymbol{\theta}|\mathbf{z}\right) = \mathrm{Pr}\left(\mathbf{z}|\boldsymbol{\theta}\right) \tag{6.4}$$

To demonstrate the derivation of the likelihood function, consider the case where the available information is a set of observed detached rocks $i = 1...n$ for a specific mountain slope, which are described by their volume v_i and the time period ΔT_z during which they occurred. Only rocks with a volume larger than v_{th} have been recorded (*th*: threshold). We make the following simplifying assumptions: a) that the rockfall follows a homogeneous Poisson process as discussed earlier and b) that the observations are free of error (i.e., all rocks are recorded). These assumptions hold under particular circumstances only, yet they are a reasonable approximation to many real situations and they are suitable for illustrative purposes. Under these assumptions, the probability of observing exactly n rocks with a volume larger than v_{th} is given by the Poisson distribution with parameter $H_V(v_{th}|\boldsymbol{\theta})\Delta T_z$ as:

$$\mathrm{Pr}\left(n|\boldsymbol{\theta}\right) = \frac{\left[H_V\left(v_{th}|\boldsymbol{\theta}\right)\Delta T_z\right]^n}{n!}\exp\left[-H_V\left(v_{th}|\boldsymbol{\theta}\right)\Delta T_z\right] \tag{6.5}$$

Given that a rock with volume larger than v_{th} has detached, the likelihood of its volume being v_i is proportional to $h_V(v_i|\boldsymbol{\theta})/H_V(v_{th}|\boldsymbol{\theta})$ for $v_i \leq v_{th}$. Because all observations are assumed independent events, the likelihood function is obtained by multiplying these terms. The likelihood function representing the observation of n rocks with volumes $v_1...v_n$ on the considered mountain area is then:

$$L\left(\boldsymbol{\theta}\,\middle|\,\mathbf{z}\right) \propto \exp\left[-H_V\left(v_{th}\,\middle|\,\boldsymbol{\theta}\right)\Delta T_z\right]\prod_{i=1}^{n} h_V\left(v_i\,\middle|\,\boldsymbol{\theta}\right) \tag{6.6}$$

$h_V(v_i|\boldsymbol{\theta})$ is the annual frequency density of V. Note that the observations apparently must relate to the frequency density and not the probability density, because we cannot observe just the largest rock that has fallen during a certain period, rather, the observed rocks may all be from the same time period.

6.4   UNCERTAINTIES IN ROCKFALL TRAJECTORY

Once a rock is released, its trajectory is mainly determined by the topography, its mode of motion (free fall, rolling bouncing or sliding) and the characteristics of the surfaces of the rock and the ground. All these factors contribute to the uncertainty in the prediction of the trajectory. Existing numerical tools model this uncertainty by means of crude Monte Carlo simulation (MCS); an overview is provided by Guzzetti et al. (2002). There exist two- or three-dimensional models and there are differences in the physical representation of the rock: The so called lumped mass approach represents the rock by a single mass point, neglecting the geometry of the stone. The rigid body approach models the stone by idealized geometries (e.g., cylinders, spheres or a cuboidal shape, Ettlin 2006) with varying physical and material properties. Hybrid models combine a lumped mass approach to simulate the free fall with a rigid body approach to simulate the contact with the ground surface. Finally, different models are used to simulate the impact of the rock on the ground (Dorren, 2003), a simple approach being the use of coefficients of restitution (Stevens 1998). The impact is the most intricate part of the falling process and its modelling is associated with large uncertainties. The modelling cannot account for the variability in the ground material (particularly in zones covered with vegetation) and the local geometry of the ground and the rock. These uncertainties are inherent to the model and can therefore be considered as aleatory. In addition, there is an epistemic uncertainty because of the limited basis for estimating the model parameters (see e.g., Robotham et al. (1995), Azzoni et al. (1995) and Chau et al. (2002) for estimation of coefficients of restitution). Additional epistemic uncertainty is due to the simplified modelling of the slope profile at the impact location. In many applications, the profile surface in the models is generated from a digital elevation model (DEM) with limited resolution and between the points provided by the DEM the terrain is assumed to be linear. If the model is 2-dimensional, the reduction to a single plane is an additional source of epistemic uncertainty.

The outcome of a two-dimensional rockfall model is illustrated in Figure 6.2. In this example, the relevant numerical result that will be utilized for risk assessment is the probability density function (PDF) of the energy of the rocks when reaching the road. This distribution should be evaluated conditional on the rock volume, $f_E(e|v)$, for different values of v. This can then be combined with the distribution representing the rock detachment. Available rockfall analysis software typically allow entering the detached volume as a Normal distributed random variable, but because the volume of detached rocks is generally not Normal distributed, results obtained with this assumption cannot be used for risk assessment directly.
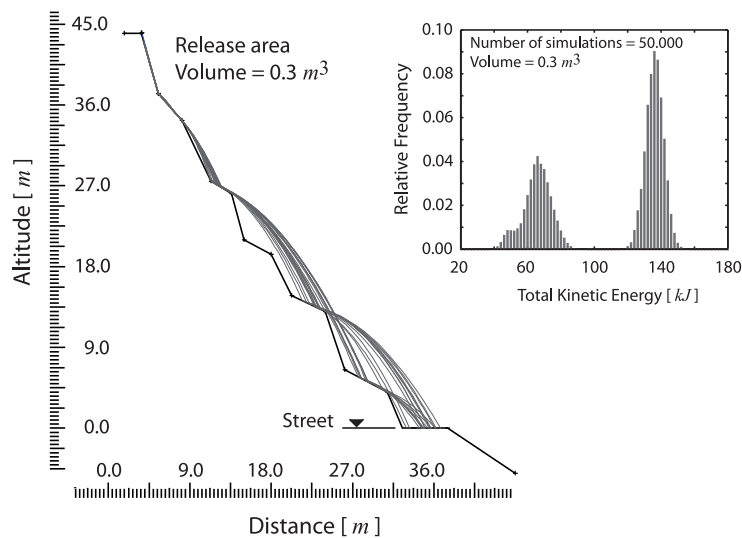
Figure 6.2 Illustration of the rockfall trajectory modelling.

MCS in existing rockfall trajectory analysis tools accounts only for the aleatory uncertainty. However, while it is important to be aware of the additional epistemic uncertainty associated with these models, for most practical applications, the error associated with neglecting this uncertainty is tolerable. This is due to the fact that in the analysis of rockfall trajectories, unlike in the modelling of rockfall exposure, the probability of extreme events is of less importance, and that the middle range of the distribution is less affected by the epistemic uncertainties.

6.5   UNCERTAINTY IN THE PERFORMANCE OF ROCKFALL PROTECTION STRUCTURES

Rockfall protection structures such as flexible nets or fixed galleries can stop the rocks, but their capacity is limited. This capacity, denoted by R, can be quantified in terms of the amount of energy that the structure can absorb. R depends on the type of structure, but also on the characteristics of the rock beyond the impact energy. The uncertainty in the capacity is considered by modelling R as a random variable, represented by its PDF conditional on the rock volume, $f_R(r|v)$. Hereby, the velocity of the rock at the impact is determined as a function of the energy and the volume. $f_R(r|v)$ should include both epistemic and aleatory uncertainty related to the structural capacity. Structural reliability analysis can be used to evaluate $f_R(r|v)$ for a given type of structure, Schubert et al. (2005). Alternatively, for standard protection systems, $f_R(r|v)$ can also be estimated from tests. However, because of their cost, the number of tests is often limited and, therefore, test results should be combined with a reliability analysis to obtain a probabilistic estimate of the capacity.

6.6   UNCERTAINTIES IN ROCKFALL ROBUSTNESS

A measure of how a system such as a rockfall protection structure reacts to a hazard or a damage or failure event can be expressed as the robustness of the system. The robustness of such a system can be accounted for by estimating the expected consequences for a given failure event following the approach described in JCSS (2008). As an example, the expected number of fatalities and injuries is evaluated by multiplying the probability that a number of people are present at the location at the time of a rockfall with the probability that somebody is killed or injured by the rock. Those probabilities represent aleatory uncertainties. There is an uncertainty as to the values of these probabilities, which is of an epistemic nature (it could be reduced by collecting additional data), but because only the expected number of fatalities and injuries enters the computation, the computed risk generally will not be very sensitive to these epistemic uncertainties. In most instances they can be neglected, as is done in practice.
An important part of system robustness modelling is the assessment of so-called "user costs", representing the socio-economical costs inflicted by the temporary disuse of the considered system, typically a transportation link. The user costs as assessed by road authorities exhibit large differences (e.g. Nash, 2003). However, it must not be concluded that these differences are due to epistemic uncertainty; rather, they are caused by different model assumptions. Therefore, this problem must be addressed by the decision maker, who must determine the model assumptions that represent his/her preferences.

7   SUMMARY AND CONCLUSIONS

A combination of inherent, modelling and statistical uncertainties needs to be dealt with in any typical decision making problem. This deliverable report provides a review of the state-of-the-art in dealing with uncertainties in the modelling, prediction, and decision-making processes. The different types and sources of uncertainties that are encountered in modelling, prediction and decision making are covered in Chapter 2 of this report. Chapter 3 gives generic guidelines for selecting the appropriate method for the treatment of uncertainties. The guidelines give information on which techniques can be used for the formulation of uncertainty for input parameters and which methods are applicable to propagate the uncertainty from input to output parameters. A brief review of the most relevant techniques and propagation methods is given in Chapter 4. Chapter 5 contains a description of methods to deal with uncertainty in decision making; here, an engineering interpretation of uncertainty is first provided with the purpose of providing a basis for the evaluation of the consistency and appropriateness of the probabilistic modelling as applied in engineering decision analysis. Thereafter a summary presentation of the prior, the posterior and the pre-posterior Bayesian decision analysis is provided given, followed by an outline of the consistent treatment of uncertainties in probability updating problems as encountered in posterior and pre-posterior decision analysis. Finally, an example on the modelling and management of uncertainties associated with rockfall hazards following a Bayesian approach is provided in Chapter 6.

Even though in many cases it is not absolutely necessary to consider the detailed characteristics of the uncertainties prevailing in decision problems, it is always useful and instructive to think through the process how uncertainties fundamentally change characteristics as function of both the point in time where they are looked upon and as function of the "scale" of the modelling used to represent them. Understanding is a prerequisite for consistent treatment of uncertainties. As an example - choosing a complicated model with many parameters for the description of uncertain phenomena may result in models with significant epistemic (statistical) uncertainties as opposed to more simple models dominated by aleatory and epistemic (model) uncertainties. The model dominated by epistemic uncertainties has the potential for reducing the uncertainties through updating following a Bayesian approach; however this should not be taken to imply that a complicated model with many uncertain parameters (hence dominated by epistemic uncertainty) should be chosen over a simpler model dominated by aleatory uncertainty in all cases. The advantage offered in the former case may be overridden by other considerations such as the objectives and context of the analysis and budget constraints. A holistic approach considering all relevant aspects of the decision problem at hand and treatment of the underlying uncertainties in a rational manner is hence advised.

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