FINAL REPORT

Numerical modelling of short-term and long-term track response

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Final report of ECP research

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1 Introduction

The role of Ecole Centrale de Paris (ECP) in the SUPERTRACK project concerns the management of the work package 3 (WP3) whose objective is to investigate the long-term dynamic behaviour of ballasted railway tracks by means of numerical simulations. This work can be decomposed into two main tasks:

1) the numerical simulation of the in-situ measurements, including the modelling of the short- and long-term behaviour of the ballasted track (Section I and II),

2) the numerical simulation of the laboratory tests (Section III).

This document corresponds to the final report and presents a global view of the ECP contributions in the SUPERTRACK project. In the first paragraph, the numerical simulation of the in-situ measurements is dealt with. At first, the short-term dynamic modelling, which consists in computing the elastic dynamic stresses in the track, is presented. A three-dimensional dynamic soil-track interaction model accounting for the periodicity of the system is proposed. The simulations are compared with the measurements performed by SCNF at the Beugnâtre site. The second paragraph is devoted to the long-term non-linear modelling which concerns the irreversible settlements of the ballast layer after a great number of cycles (train passages). The methodology is presented and parameters of the non-linear model are updated using triaxial tests performed by NGI. Finally, the long-term response of the ballasted layer is presented.

In the third part, the numerical simulation of the laboratory tests, that is the track-box, are presented. A description of the track-box is given and the response of this system to vertical loadings (which represent the train passages) is simulated and analysed.

2 Numerical simulation of the in-situ measurements: the short-term dynamic modelling

2.1 Soil-track model

In this section, the three-dimensional model for the soil-track system is presented. This model is based on a geometrical periodic formulation and takes into account the dynamic soil-structure interaction.

2.1.1 Periodic model

The proposed model takes advantage of the spatial periodicity of the track-soil system. In this manner, one can reduce the analysis for the overall system to a problem posed on a generic cell (see Figs. 1&2 in which only the track-structure is represented).
Mathematical formulation of the periodic model

The periodic formulation is constructed by using some classical results due to Floquet [10]. For the sake of clarity, only the one-dimensional case is presented. Let $A$ be a differential operator with periodic coefficients ($L$ being the period) such as

$$A(x+L)u = A(x)u, \quad (1)$$

for any real $x$ and for any function $u$ belonging to its domain of definition $D(A)$. Moreover, let us introduce the family of operators $A_k'$ as the restriction of $A$ on a reference cell $0 < x < L$. Then, as shown in [8,9]:

If

$$A u = f, \quad (2)$$

has a unique solution $u \in D(A)$, with $f$ a function belonging to the image of $A$, and if

$$A_k' u' = f' \quad (3)$$

has a unique solution $u' \in D(A_k')$ such as

$$u'(L) = e^{i\kappa L} u'(0) \quad \forall \kappa \in \left[ -\pi/L, \pi/L \right], \quad (4)$$
then \( u' \) is to the Floquet transform of \( u \) defined by

\[
u'(x', \kappa) = \sum_{Ny=-\infty}^{+\infty} u(x' + NyL) e^{i\kappa NyL}, \tag{5}\]

for any \( x' \in ]0,L[ \) and any wavenumber \( \kappa \in ]-\pi/L, \pi/L[ \). It should be noted that Eq. (2) can represent the usual dynamic equation for a system subjected to an external force \( f \), \( u \) being the displacement field. Consequently, instead of solving Eq. (2) on infinite space, one can solve Eq. (3) on the generic cell, and one can build the solution \( u \) using the inverse Floquet transform:

\[
u(x) = \frac{1}{2\pi} \int_{-\pi/L}^{\pi/L} u'(x', \kappa) e^{-i\kappa NyL} \, d\kappa, \tag{6}\]

with \( x = x' + NyL \). These results have been extended to the three-dimensional soil-structure dynamic interaction problem with a period \( L \) along one direction [8, 9].

### 2.1.2 Soil-track interaction model

Moreover, with such a model, the dynamic soil-track interaction is taken into account (and for very long structures partially embedded in the soil, such as railway tracks, this phenomena is of great importance). For this, the computer code MISS3D [7], developed at ECP, is used. This software is based on a domain decomposition method [1, 6]. Thus, the three-dimensional domain considered (the generic cell) is decomposed into two sub-domains (the track-structure and the soil) which are coupled on a given interface (see Fig. 3). Consequently, each sub-domain can be independently modelled. For example, the boundary element method (BEM) with special Green functions is used for the soil while the track-structure is modelled by using the finite element method (FEM) and its dynamical behaviour is characterized by periodic modes.

![Fig. 3 Generic cell decomposed into two sub-domains (track & soil).](image)
Mathematical formulation of the substructure model

Some results from [1,6] are briefly recalled. Let $u_{st}'$ the displacement field in the bounded generic structure cell. It is decomposed on a given basis of periodic modes $\{\phi_k'\}_{k=1,...,N}$ which satisfy Eq. (4). Then, the displacement field $u_{st}'$ is written as

$$u_{st}'(x') = \sum_{k=1}^{N} \phi_k'(x') c_k = \phi(x') c,$$

in which $x' \in [0,L[$. Moreover, the soil displacement in the generic cell $u_s'$ is defined by

$$u_s' = u_i' + u_{do}' + u_{sc}' ,$$

with $u_i'$ the incident wave field, $u_{do}'$ the locally diffracted wave field and $u_{sc}'$ the scattered wave field. The soil displacement can be further decomposed as

$$u_s'(x') = u_i'(x') + u_{do}'(x') + \sum_{k=1}^{N} u_{dk}'(x') c_k ,$$

where $\{u_{dk}\}_{k=1,...,N}$ are elastodynamic fields. On the interface $\Gamma'$ (see Fig. 3), kinematical conditions read

$$u_i' + u_{do}' = 0 \text{ on } \Gamma',$$

$$u_{dk}' = \phi_k' \text{ on } \Gamma'. $$

Using a standard Galerkin approximation procedure in written the equilibrium of the generic structure cell in a weak sense, for any $\phi_k'$ in the basis, we obtain the following linear system

$$[K_{st}(\kappa) - \omega^2 M_{st}(\kappa) + K_s(\kappa,\omega)] c(\kappa,\omega) = F_{st}(\kappa,\omega) + F_s(\kappa,\omega),$$

for any angular frequency $\omega$ and any wavenumber $\kappa \in ]-\pi/L, \pi/L[$. Matrices $[K_{st}]$ and $[M_{st}]$ correspond respectively to the stiffness and mass matrices of the structure, $[K_s]$ is the soil impedance, $F_{st}$ is the generalized force vector applied on the structure and $F_s$ is the generalized force vector due to the incident wave field.

### 2.1.3 Dynamic response of the soil-railway track system to a constant moving-load

A constant force $f_t$ moving along the periodicity direction at a constant velocity $V$ is applied on the global railway track structure. This force is defined by ($e_z$ being the vertical ascending direction)

$$f_t(x_0,t) = F_0 \delta(x_0-X) \delta(y_0-Y-Vt) \delta(z_0-Z) e_z,$$

in which $x_0=(x_0,y_0,z_0)$ is a point of the railway track system and where $(X,Y,Z) = X$ corresponds to the position of the moving load at time $t = 0$. The parameter $F_0$ is a real
constant. It is proved \cite{12} that the displacement response in the frequency domain \( u(x_1, X, \omega) \), for any \( x_1 \) of the railway track system, due to the moving load \( f_t(x_0, t) \) defined by Eq. (13), is given by

\[
u(x_1, X, \omega) = e^{i\kappa_0 Y} F_0 \frac{1}{2\pi} \int_0^L e^{-ik_0 y'} h'(x_1, x, y', Z; \kappa_0, \omega) dy', \tag{14}\]

in which \( \kappa_0 = \omega / V \). For any \( \kappa \in \left[-\pi/L, \pi/L\right] \) and circular frequency \( \omega \) in the band of analysis, \( h'(x_1, x, y', Z; \kappa, \omega) \) corresponds to the displacement solution at the point \( x_1 \) when the railway track structure is subjected to a force \( f_t(x) = \delta(x-X) \delta(y-y') \delta(z-Z) \). It can be seen in Eq. (14) that the displacement is directly deduced from the function \( h' \), without computing the inverse Floquet transform.

2.2 Numerical example - Application to ballasted railway tracks

The soil-railway track model previously proposed is used to simulate the dynamic responses of the track at the Beugnâtre site \cite{13}.

2.2.1 Track-structure model in the generic cell

The studied track section is composed of three layers (see Fig. 4). The two first ones are constituted of ballast and sub-ballast whereas the third is made up of sand and gravels.

![Fig. 4 FE model for the track in the generic cell.](image)

Their measured mechanical and geometrical characteristics \cite{15} are given in Table I. Moreover, the measured hysteretic damping ratio for the three layers equals respectively 8\%, 6\% and 4\%. On this generic track-structure, two sleepers in concrete are taken into account and are connected in pairs by means of steely crossbars. Moreover, rails have been modelled and are connected to the sleepers with pads.
### TABLE I: MECHANICAL AND GEOMETRICAL CHARACTERISTICS OF THE TRACK.

<table>
<thead>
<tr>
<th></th>
<th>ballast</th>
<th>sub-ballast</th>
<th>sub-layer</th>
<th>crossbar</th>
<th>sleeper</th>
<th>rail</th>
<th>pad</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ (N/m²)</td>
<td>2.09 $10^9$</td>
<td>3.62 $10^9$</td>
<td>5.44 $10^9$</td>
<td>2.1 $10^9$</td>
<td>3 $10^9$</td>
<td>2.1 $10^9$</td>
<td>/</td>
</tr>
<tr>
<td>$\rho$ (kg/m³)</td>
<td>1550</td>
<td>2265</td>
<td>2270</td>
<td>7800</td>
<td>2200</td>
<td>7800</td>
<td>/</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.15</td>
<td>0.20</td>
<td>0.20</td>
<td>0.3</td>
<td>0.25</td>
<td>0.3</td>
<td>/</td>
</tr>
<tr>
<td>$k$ (N/m)</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>159 $10^6$</td>
</tr>
<tr>
<td>$h$ (m)</td>
<td>0.75</td>
<td>0.20</td>
<td>0.50</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

$E$: Young's modulus, $\rho$: mass density, $\nu$: Poisson's ratio, $k$: vertical stiffness, $h$: thickness

### 2.2.2 Soil model in the generic cell

The soil is assumed to be a stratified visco-elastic half-space. The mechanical and geometrical characteristics are given in Table II (from [15]).

### TABLE II: MECHANICAL AND GEOMETRICAL CHARACTERISTICS OF THE SOIL

<table>
<thead>
<tr>
<th></th>
<th>lime treated loam</th>
<th>clayey loam</th>
<th>loamy silt</th>
<th>loamy silt</th>
<th>loamy silt</th>
<th>silty clayey</th>
<th>fractured chalk</th>
<th>fractured chalk</th>
<th>fresh chalk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p$ (m/s)</td>
<td>935</td>
<td>224</td>
<td>468</td>
<td>281</td>
<td>187</td>
<td>224</td>
<td>281</td>
<td>2040</td>
<td>1102</td>
</tr>
<tr>
<td>$C_s$ (m/s)</td>
<td>500</td>
<td>120</td>
<td>250</td>
<td>150</td>
<td>100</td>
<td>120</td>
<td>150</td>
<td>400</td>
<td>450</td>
</tr>
<tr>
<td>$\rho$ (kg/m³)</td>
<td>1990</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>1900</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$h$ (m)</td>
<td>0.8</td>
<td>1.5</td>
<td>1.1</td>
<td>1.2</td>
<td>1.00</td>
<td>1.30</td>
<td>2.60</td>
<td>2.50</td>
<td>5.70</td>
</tr>
</tbody>
</table>

$C_p$: pressure wave velocity, $C_s$: shear wave velocity, $\rho$: mass density, $\beta$: hysteretic damping ratio, $h$: thickness

### 2.2.3 Convergence analyses

A number of $N_y=70$ cells is chosen (cf. [5]). Moreover, a number of 15 periodic modes is considered.

### 2.2.4 Linear responses due to moving loads

The soil-track system is subjected to a moving load which models a Thalys train whose speed equals 298 km/h. The first six axles (corresponding to one locomotive and the superior part of the first carriage) have been modelled. Simulations are compared with acceleration measurements performed at the Beugnâtre site (Section 2-pk140.934). For the frequency domain, measured and simulated vertical accelerations are compared respectively for sensors on the sleeper and in the sub-ballast layer (Fig. 5). Fig. 6 shows the same quantities in the time domain. It can be observed a general good fitting between the two types of result.
For nodes located in the ballast layer under the sleeper (see fig. 7), Figures 8 to 10 show the computed elastic displacements with respect to time, due to the passage of one axle.
**Fig. 8**: Displacement in the x-direction for nodes in the ballast layer, with respect to time. Global view (left) and Zoom (right).

**Fig. 9**: Displacement in the y-direction for nodes in the ballast layer, with respect to time. Global view (left) and Zoom (right).
Once the displacements have been calculated, the elastic stress states can be evaluated. For the three elements of the ballast layer under the sleeper (see Fig. 7), the evolution with time of the stresses is presented in Figures 11 to 13.

**Fig. 10**: Displacement in the z-direction for nodes in the ballast layer, with respect to time. Global view (left) and Zoom (right).

**Fig. 11**: Stress state in the element at the bottom of the ballast layer.
The presented stress states are used as reference cycles (associated to one train axle) for the non-linear calculus and then will be repeated one million times to get the irreversible displacements of the ballast layer.
3 Numerical simulation of in-situ measurements: long-term dynamic modelling

The methodology used in [4] is briefly recalled. The characteristics of the problem studied are the three-dimensional geometry and the non-linear irreversible behaviour of the ballast. Unfortunately, the 3D non-linear dynamic finite element analysis of such a problem is not yet to be envisaged. Here, it is suggested to decompose the problem into two parts in which the linear and non-linear effects are uncoupled. Then, the overall behaviour is considered not to be far from a linear one so that the global effects applying on several meters of the track can be considered to be linear elastic and calculated with Miss3D [7] (see Section I), while the non-linearities have only a local effect and can be studied on a given section with the Gefdyn software [3] (Section II).

3.1 Mathematical formulation

Let Ω be the unbounded domain of which forces \( f \) (moving load) apply. Under small strain and small displacement assumption, the displacement field \( u \) is such that

\[
\text{div} \sigma(u) + f = \rho \partial_t^2 u \quad \text{in } \Omega,
\]

with initial conditions. The tensor \( \sigma(u) \) is the stress tensor which is supposed to present a small non-linearity with respect to \( u \) such that it can be expressed by

\[
\sigma(u) = \sigma_l(u) + \sigma_n(u)
\]

where \( \sigma_l(u) \) is the linear part and \( \sigma_n(u) \) is small with respect to \( \sigma(u) \).

![Schematization of the global and local scales](image)

If \( \Omega' \) presents a limited zone on which the non-linear analysis is to be performed, we can consider that the behaviour is linear outside this area and its interaction with \( \Omega \) can be
considered by appropriate boundary conditions. A difficulty is risen as the support of \( f \) is defines on a domain much larger than \( \Omega' \).

Let consider the displacement field \( u_o \) which is the solution of the large scale elastic problem

\[
div \sigma(u) + f = \rho \partial_t u_o \quad \text{in} \, \Omega.
\]  
(18)

The restriction of this equation to \( \Omega' \) is used to define the equivalent loading \( f' \) in this domain

\[
f' = -div \sigma(u_o) + \rho \partial_t u_o \quad \text{in} \, \Omega'.
\]  
(19)

This equivalent loading can be used in the local model instead of \( f \)

\[
div \sigma(u) + f' = \rho \partial_t u \quad \text{in} \, \Omega'.
\]

### 3.2 Modelling of the non linear behaviour

The ECP’s elastoplastic multi-mechanism model, commonly called Hujeux model ([2,11]) is used to represent the soil behaviour. This model can take into account the soil behaviour in a large range of deformations. The model is written in terms of effective stress. The representation of all irreversible phenomena is made by four coupled elementary plastic mechanisms: three plane-strain deviatoric plastic deformation mechanisms in three orthogonal planes and an isotropic one.

The model uses a Coulomb type failure criterion and the critical state concept. The evolution of hardening is based on the plastic strain (deviatoric and volumetric strain for the deviatoric mechanisms and volumetric strain for the isotropic one). To take into account the cyclic behaviour a kinematical hardening based on the state variables at the last load reversal is used.

The model is written in the framework of the incremental plasticity, which assumes the decomposition of the total strain increment in two, elastic and plastic, parts.

The elastic part is supposed to obey a non-linear elasticity behaviour, where the bulk (\( K \)) and the shear (\( G \)) moduli are functions of the mean effective stress (\( p' \))

\[
K = K_{ref} \left( \frac{p'}{p_{ref}} \right)^{n_e} \quad \text{and} \quad G = G_{ref} \left( \frac{p'}{p_{ref}} \right)^{n_e}
\]

\( K_{ref} \) and \( G_{ref} \) are the bulk and shear moduli measured at the mean reference pressure (\( p_{ref} \)).

Adopting the soil mechanics sign convention (compression positive), the deviatoric yield surface of the \( k \) plane does only concern stresses on the \( (i,j) \) plane:

\[
\sigma_{k_{ij}} = \sigma_{i} c_{i} \otimes c_{i} + \sigma_{j} c_{j} \otimes c_{j} + \sigma_{ij} c_{i} \otimes c_{j}
\]

with:
Thus, the deviatoric primary (m) yield function of the k plane is given by:

$$f_k^m (\sigma, \varepsilon^p, r_k) = q_k^m - \sin \phi'_{pp} \cdot p_k' \cdot F_k \cdot r_k^m \quad k \in [1, 2, 3]$$

with:

$$F_k = 1 - b \ln \left( \frac{p_k'}{p_c} \right)$$

$$p_c = p_{co} \exp (\beta \varepsilon^p_v)$$

where, $\phi'_{pp}$ is the friction angle at the critical state. The parameter $b$ controls the form of the yield surface in the $(p', q)$ plane and varies from $b = 0$ to $1$ passing from a Coulomb type surface to a Cam-Clay type one. $\beta$ is the plasticity compression modulus and $p_{co}$ represents the critical state stress corresponding to the initial voids ratio.

The internal variable $r_k$, called degree of mobilized friction, is associated with the plastic deviatoric strain. This variable introduces the effect of shear hardening of the soil and permits the decomposition of the behaviour domain into pseudo-elastic, hysteretic and mobilized domains, it is given by

$$r_k = r_{k,el} + \frac{\int_{0}^{t} \varepsilon^p dt}{a + \int_{0}^{t} \varepsilon^p dt}$$

with

$$a = a_1 + (a_2 - a_1) \alpha_k (r_k)$$

where

$$\alpha_k = \begin{cases} 0 & \text{if } r_{k,clas} < r_k < r_{k,hyg} \\ \left( \frac{r_k - r_{k,hyg}}{r_{k,mob} - r_{k,hyg}} \right)^m & \text{if } r_{k,hyg} < r < r_{k,mo} \\ 1 & \text{if } r_{k,mo} < r < 1 \end{cases}$$

$a_1$, $a_2$ and $m$ are model parameters controlling the evolution of the shear with plastic deformation. $r_{k,hyg}$ and $r_{k,mo}$ designate the extend of the domain where hysteresis degradation occurs. The $r_{k,hyg}$ marks the extend of the domain where no volume variation takes place and corresponds to the so-called volumetric threshold.
The cyclic yield surface is given by:

\[ f_k^{cyclic}(\sigma, \varepsilon^p, r_k) = q_k^c - \sin \phi'_{pp} \cdot F_k \cdot r_k^c \quad k \in [1, 2, 3] \]

where \( q_k^c \) is the deviatoric stress with respect to the last unloading point:

\[ q_k^c = \left( \frac{1}{2} \text{Tr}(s_k^c \cdot s_k^c) \right)^{1/2} \]

\[ s_k^c = s_k - (T_k^h - \frac{h_k}{q_k^c} p_k^h F_k \sin \phi'_{pp}) \]

with:

\[ T_k^h = -\frac{s_k^h}{p_k^h F_k \sin \phi'_{pp}} \]

\[ \frac{h_k}{q_k^c} = \frac{s_k^h}{q_k^c} \]

The isotropic yield surface is assumed to be:

\[ f_{iso} = |p'| - d p_c r_{iso} \]

with:

\[ r_{iso} = r_{iso}^{\text{ela}} + \frac{\int_0^t |(\varepsilon_p)^{iso}| dt}{c \left( p_{ref} + \int_0^t |(\varepsilon_p)^{iso}| dt \right)} \]

where \( d \) is a model parameter representing the distance between the isotropic consolidation line and the critical state line in the (\( e \), \( \ln p' \)) plane and \( c \) controls the volumetric hardening.

In the model, an associated flow rule in the deviatoric plane (k) is assumed, and the Roscoe’s dilatancy law is used to obtain the increment of the volumetric plastic strain of each deviatoric mechanism so that:

\[ \varepsilon_{vpk} = \dot{\lambda}_k^p \cdot \alpha_{\psi} \cdot \alpha_k(r_k^c) \left( \sin \psi - \frac{q_k}{p_k} \right) \]

where \( \psi \) is the characteristic angle and \( \alpha_{\psi} \) a constant parameter.
The parameters of the model can be classified with respect to the way of their identification. They are given in Table III.

Table III Classification of the Elastoplastic model parameters

<table>
<thead>
<tr>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directly measured</td>
</tr>
<tr>
<td>$K_{ref}$, $G_{ref}$</td>
</tr>
<tr>
<td>Critical State and Plasticity</td>
</tr>
<tr>
<td>$q_{p0}$, $\beta$</td>
</tr>
<tr>
<td>$p_{eo}$, $d$</td>
</tr>
<tr>
<td>Flow Rule and Isotropic hardening</td>
</tr>
<tr>
<td>$\psi$</td>
</tr>
<tr>
<td>Threshold domains</td>
</tr>
<tr>
<td>$r_{ela}$, $r_{leps}$</td>
</tr>
</tbody>
</table>

3.3 Model parameters identification

Two triaxial tests have been performed on the ballast by NGI. The two samples with different initial unit weights (Test 1: 1550 Kg/m³ and Test2: 1621 Kg/m³) have been subjected to cyclic $q/p$ constant stress paths during 100000 cycles. The samples are then monotonically deformed up to 14% vertical strain. The confining pressures $p_{o}'$ is the same in the two tests (40 kPa). The obtained results are given in Figures 15 and 16. The following aspects are noticed on these results:

- The radial strains are negligible in the beginning of the tests for axial strains up to 0.18%
- The irreversible volumetric strain is negligible during the reloading paths
- The q-Epsz curves present a very steep (almost infinite) slope at the beginning of each unloading (Figure 17).

The first phenomenon can be either due to the anisotropy of the initial state or due to the performance of the radial measurement ring, or even both. Even though it is possible to take into account the initial anisotropy of the material by the model, we have not tried to investigate this aspect as it is not relevant for our actual objective. The second remark, encourage us to suppose that the isotropic mechanism is probably inactive as no significant volumetric plastic deformation is obtained during the cyclic loading. As there is no evidence of such a plastic deformation mechanism, and this hypothesis, reduces the number of the parameters for which no specific measurement is available and the only was of identifying the associated parameters would have been by curve fitting, we consider that this mechanism is
inactive. The last phenomenon which can be attributed to the relaxation of the contact forces at each unloading, is not integrated in the constitutive model. Thus, we have decided to model the overall behaviour of each loop in our simulations.

**Fig. 15:** Observed behaviour of the experiment Test1 on the ballast obtained by NGI

**Fig. 16:** Observed behaviour of the experiment Test2 on the ballast obtained by NGI
The strategy to choose the model parameters has been to fix the measurable parameters using the monotonic response of the model on the two samples. This has resulted in the following values:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test 1</th>
<th>Test 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{ref}$ (MPa)</td>
<td>112</td>
<td>112</td>
</tr>
<tr>
<td>$G_{ref}$ (MPa)</td>
<td>83</td>
<td>83</td>
</tr>
<tr>
<td>$n_e$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$p_{ref}$ (MPa)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\phi_{pp}$ ($^\circ$)</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>$\beta$</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>$b$</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>$p_{co}$ (MPa)</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>$\psi$ ($^\circ$)</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The hardening parameters $a_1$ and $a_2$ have been chosen to obtain a good fit of the overall form of the $q$-$\varepsilon$ curves and loops. This resulted in decreasing the $a_2$ variable with the number of cycles. We obtained the evolution of $a_2$ parameter with the cumulative incremental work.
(Σdσ:de) given in Figure 18. \( r_{\text{hys}} \) was chosen to optimise the volumetric strain evolution. From the initial state to 200 cycles it increases from 0.03 to 0.21, but it stays constant afterwards.

\[
\begin{array}{c|c|c}
\text{Test 1} & \text{Test 2} \\
\hline
m & 1.0 & 1.0 \\
\varepsilon^{\text{sta}} & 0.005 & 0.005 \\
\varepsilon_{\text{hys}} & 0.03-0.21 & 0.03-0.21 \\
\rho_{\text{mob}} & 0.8 & 0.8 \\
\end{array}
\]

Figures 19 and 20 show the response of the model for the first cycles of loading compared to the experimental results. The overall behaviour is well modelled, though the volumetric and axial strains are underestimated. However, it is worth noting that the magnitude of simulated strain variation during one cycle is comparable to the experimental observations and they have almost the same form. The evolution of the form of the stress-strain loops is given in Figures 21 and 22. The variation of the axial deformations with the number of loading-unloading cycles is given in Figure 23. It should be signalled that the only parameter that is different in the two tests is the \( p'c_0 \) which depends on the initial void ratio (density).

The static response of the two tests after 10E5 loading-unloading cycles is given in Figures 24 and 25. The static monotonic response of the samples without ongoing any cycling loading simulated by the model is also represented (dashed lines). As it can be noticed on these figures, the cyclic loading results in compaction of the material which increases its resistance. The simulated response of Test 1 presents a peak which is not observed experimentally. The same type of behaviour is obtained by Suiker [14] on sub-ballast experimentally (Figure 26).
Fig. 19: Comparison between simulated (red) q/p constant test and experimental (green) Test 1 on ballast, $q - \varepsilon$ and $\varepsilon, - \varepsilon$ planes.
Fig. 20: Comparison between simulated (magenta) $q/p$ constant test and experimental (blue) test 1 on ballast, $q - \varepsilon_t$ and $\varepsilon_v - \varepsilon_t$ planes.
Fig. 21: Comparison between simulated (magenta) q/p constant test and experimental (blue) Test 1 on ballast, $q - \varepsilon_i$ and $\varepsilon_v - \varepsilon_i$ planes.

Fig. 22: Comparison between simulated (red) q/p constant test and experimental (green) Test 2 on ballast, $q - \varepsilon_i$ and $\varepsilon_v - \varepsilon_i$ planes.
Fig. 23: Evolution of the axial deformation with the loading-unloading cycles
Numerical simulation: red & magenta
Experimental results: green & blue
Fig. 24: Comparison between simulated (red) and experimental (green) results of the final stage of Test 1 on ballast.

Dashed red: Only monotonic loading simulated
Solid red: cyclic loading followed by monotonic loading simulated
$q - \varepsilon$ and $\varepsilon_v - \varepsilon$ planes.
Fig. 25: Comparison between simulated (blue) and experimental (magenta) results of the final stage of Test 2 on ballast.

Dashed blue: Only monotonic loading simulated
Solid blue: Cyclic loading followed by monotonic loading simulated

$q - \varepsilon_t$ and $\varepsilon_v - \varepsilon_t$ planes.
3.4 Estimation of the irreversible deformations of the track

Once the model parameters identified, stress paths similar to those encountered under the rail in the track during the passage of one axle under isotropic elastic hypotheses have been simulated and the evolution of the vertical deformation with the number of the loading-unloading cycles up to 1 million cycles has been studied. In order to prevent the irreversible horizontal deformations in the direction of the track, plane strain condition in this direction has been applied. The computed $q=\sigma_z-\sigma_y, p=\text{tr}(\sigma)/3$ stresses during the application of the cycles is given in Figure 27. As it can be noted, the change is not very important.
It should be noted that the same parameters as the ones used while modelling the triaxial tests have been applied. As the \( a_2 \) parameter varies with the cumulative incremental work, the evolution of this quantity in triaxial tests and in the track stress path simulations are given in Figure 28. It can be noticed that less work is cumulated while applying the track loading path. The predicted variation of the vertical strain with the number of loading cycles with the plane strain hypothesis and when applying the whole elastic stress tensor are given in Figures 29 and 30. The following remarks can be made:

- the evolution of the vertical strains with the number of cycles can be divided into several parts. An initial important evolution, followed by a less increasing phase which ends up to an accelerated evolution at high number of cycles.
- The variation of the irreversible vertical deformation changes with the number of cycles. In the table below the value of the vertical deformation for each loading unloading cycle and the permanent deformation at the end of each cycle for different number of cycles are given.
- The vertical strains while applying the 3D stress paths are much higher than those obtained with the plane strain hypothesis.
Number of cycles | $\Delta \varepsilon / dN$ | $\delta \varepsilon_p / dN$ | $\varepsilon_z$
--- | --- | --- | ---
1E6 | 4.3946e-004 | 4.9300e-006 | 0.0188
1E5 | 4.3944e-004 | 1.2000e-008 | 0.0067
1E4 | 4.4439e-004 | 1.4000e-008 | 0.0055
1E3 | 4.3945e-004 | 4.9280e-006 | 0.0053
100 | 4.4907e-004 | 1.1641e-005 | 0.0024

*Fig. 28: Evolution of the cumulated work with the number of cycles in the two stress paths*
Fig. 29: Evolution of the vertical strain with the number of cycles under track stress path (plane strain condition)

Fig. 30: Evolution of the vertical strain with the number of cycles under track stress path (complete stress path)
CONCLUSIONS AND PERSPECTIVES

The results obtained in triaxial tests done by NGI on large ballast samples have been used to identify the parameters of the ECP’s elastoplastic model. The monotonic results have given almost all the parameters except the hardening law which has been identified using the cyclic test results. Moreover, the irreversible deformations of the sample ballast in a track have been estimated by using the stress paths to which the ballast will be submitted during the passage of a moving loads corresponding to one axle during 1 million cycles. These stress paths have been obtained with an elastic homogeneous hypothesis behavior. It was noticed that fixing the horizontal deformations of the track would give more realistic results on important number of cycles.

REFERENCES


Appendix A

MANUAL FOR THE SOFTWARE
A- LINEAR CALCULUS

This part is devoted to the software built for the calculation of the linear responses of the railway track due to moving loads. This tool is based on the use of the SDTools (a toolbox in the Matlab environment: http://www.sdtools.com/) and the MISS software (http://www.mssmat.ecp.fr/rubrique.php3?id_rubrique=18).

Three directories are used: PRE-CALCULUS, MISS-CALCULUS AND POST-CALCULUS. The first directory contains the files necessary for the construction of the inputs for the MISS calculus. The MISS part is devoted to the calculation of the soil impedance. Finally, PRE-CALCULUS concerns the files used for the calculation of the linear responses of the track due to the passage of moving loads.

A.1 PRE-CALCULUS

main.m: file to be run. This file is divided into several parts:

constr_data: built the FE model of the track. Rails and pads are taken into account. The related mass and stiffness matrices are obtained.

call load_modelEF.m get model_EF.mat

calcul_mode: calculate the periodic modes.

call cal_modeperiodicI.m get modes.mat

extract_interface: built the FE model of the soil-railway track interface.

miss_files: write files input for the Miss computation.

call miss_write.m get track.miss (mesh file) and track.chp (mode file)

A.2 MISS CALCULUS

INPUTS

Miss.in: file to be run.

track.miss and track.chp: files previously built.
OUTPUTS

track.IMPDC : binary file containing the soil impedance.

track.CTR : binary file containing solutions at chosen observation points in the soil.

A.3 POST-CALCULUS

main.m : file to be run. This file is divided into several parts :

data_load: load data related to the FE model, periodic modes, the frequency band of analysis and the wavenumber-slowness range.

read_ks: read the file containing the soil-structure impedance.

call read_imp.m → get soil_imp.mat

read_uc: read the file containing solutions at the observation points.

call read_controlpt.m → get controle_pts.mat

build_kappa: construction of the reduced dynamic stiffness matrix of the track structure.

call cal_kappa.m → get kappa.mat

cal_ref: calculate solution (frequency response function) of the soil-structure dynamic equation for the reference cell.

call resol.m

cal_syst: calculate (frequency and time) responses due to a hammer impact or moving force for the overall domain.

call hammer_force.m or moving_force.m → get vector MatCU

To obtain the linear responses of the track due to the passage of several axles,
call multiple_axle.m.
B- NON-LINEAR CALCULUS

This part is devoted to the software built for the simulation of the long-term behaviour of the ballasted railway tracks. This tool is based on the use of the SDTools (a toolbox in the Matlab environment: http://www.sdtools.com/) and the GEFDYN software (http://www.mssmat.ecp.fr/rubrique.php3?id_rubrique=16)

Three directories are used: PRE-CALCULUS, GEFDYN-CALCULUS and POST-CALCULUS.

B.1 PRE-CALCULUS

built_modelEFsym.m: file to be run. This file is divided into several parts:

built_modelsym: built the FE model for the non-linear calculus, that is to say the symmetrical part of the reference cell.

→ get modelEF_gefdyn.mat

built Disp: built the associated displacement vector.

→ get displacement_time_gefdyn.mat

B.2 GEFDYN CALCULUS

This calculus is devoted to the evaluation of the linear stress state in the ballast layer.

INPUTS

ballast_elastique.in: file to be run (contains data of the elastic constitutive law and of the loading).

ballast_elastique.geom: file with the mesh data.

OUTPUTS

ballast_elastique_SAVE: file with the stress state at given points of the ballast layer.
B.3 POST-CALCULUS

call write_lin_materiau.m  get pt.lin (file with data of the non-linear constitutive law obtained from triaxial tests).

call write_chargt_materiau.m  get pt.chr (file with the elastic stress state).

The Lawyer software is here used with the input files pt.lin and pt.chr.

get pt_resloi_01 (file with the plastic strains and other relevant quantities)

To read it, call read_resloi.m .