Grant Agreement No.: 226479

# SafeLand

## Living with landslide risk in Europe: Assessment, effects of global change, and risk management strategies

7th Framework Programme
Cooperation Theme 6 Environment (including climate change)
Sub-Activity 6.1.3 Natural Hazards

**Deliverable D5.4**

Quantification of uncertainties in the risk assessment and management process

Deliverable/Work Package Leader: ETHZ

Revision: 1 - [Final]

July 2011

| **Rev.** | **Deliverable Responsible** | **Controlled by** | **Date** |
|---|---|---|---|
| 0 | ETHZ | ICG | 30.06.2011 |
| 1 | ETHZ | ICG | 08.07.2011 |
| | | | |

# SUMMARY

The consideration and treatment of uncertainties is an essential part of any risk assessment and risk management process. Uncertainties can either be naturally inherent or modelling and statistical related. This deliverable aims to provide a rational basis for the quantification of the various uncertainties existent in the risk assessment and risk management processes. A general description of the uncertainties in the modelling, prediction and decision making processes and their treatment and propagation can be found in Deliverable D0.3 of the SafeLand project. In order to obtain a complete picture of the issues and aspects concerning the treatment, quantification and management of uncertainties in the risk assessment, risk management and decision making processes, it is advised that this deliverable report be read in conjunction with the report of Deliverable D0.3.

The issue of knowledge and uncertainty in the real world decision making platform is introduced in Chapter 2 of this report. Here, a differentiation of uncertainties into aleatory and epistemic uncertainties is introduced, primarily for the purpose of setting focus on how uncertainty may be reduced. In Chapter 3, focus is directed on the uncertainties in the different models used for the quantification of risk and the characterisation of parameters in the models. Guiding principles and a general basis for the modelling and representation of the underlying uncertainties in the use of these models for the quantification and estimation of risks are provided. A Bayesian approach is advocated for the representation, handling and management of uncertainties in the context of decision making and is described in Chapter 4. Finally, an example on the modelling and management of uncertainties associated with rockfall hazards following a Bayesian approach is provided in Chapter 5.


# Note about contributors

The following organisations contributed to the work described in this deliverable:


**Lead partner responsible for the deliverable:**
ETHZ

*Deliverable prepared by:*
Harikrishna Narasimhan and Michael Faber

**Partner responsible for quality control:**
ICG

*Deliverable reviewed by:*
Farrokh Nadim

**Other contributors:**
ICG
AMRA
AUTH
GeoZS

# CONTENTS

# 1 INTRODUCTION

## 1.1 BACKGROUND

Risk assessment and risk management can be seen as essential and integral aspects and inputs to the decision planning, decision support and decision making processes. The importance of risk is brought out in the following quotation of Henry Ford : “The best we can do is size up the chances, calculate the risks involved, estimate our ability to deal with them, and then make our plans with confidence.” Decision making in general is a difficult issue due to the significant underlying uncertainties and complex interrelation of events and choices affecting the benefits and losses associated with decisions. Typical decision problems are subject to a combination of inherent, modelling and statistical uncertainties. Estimates of risk are hence pervaded by significant uncertainty due to the uncertainty in data and indicators, and uncertainty in models which use data and indicators as inputs. Neglecting uncertainties could lead to an unsafe estimate of loss, thereby hindering the desired reduction of risk to acceptable levels, or to an overestimation of risk, resulting in uneconomic mitigation countermeasures.

This deliverable aims to provide a rational basis for the quantification of the various uncertainties existent in the risk assessment and risk management processes. Focus is directed on the different models used for the quantification of risk and the characterisation of parameters in the models; a Bayesian approach is advocated for the representation, handling and management of uncertainties. A general description of the uncertainties in the modelling, prediction and decision making processes and their treatment and propagation can be found in Deliverable D0.3 of the SafeLand project. In order to obtain a complete picture of the issues and aspects concerning the treatment, quantification and management of uncertainties in the risk assessment, risk management and decision making processes, it is advised that this deliverable report be read in conjunction with the report of Deliverable D0.3.

## 1.2 STRUCTURE OF THIS DELIVERABLE

The issue of knowledge and uncertainty in the real world decision making platform is introduced in Chapter 2 of this report. Here, a differentiation of uncertainties into aleatory and epistemic uncertainties is introduced, primarily for the purpose of setting focus on how uncertainty may be reduced. In Chapter 3, focus is directed on the uncertainties in the different models used for the quantification of risk and the characterisation of parameters in the models. Guiding principles and a general basis for the modelling and representation of the underlying uncertainties in the use of these models for the quantification and estimation of risks are provided. A Bayesian approach is advocated for the representation, handling and management of uncertainties in the context of decision making and is described in Chapter 4. Finally, an example on the modelling and management of uncertainties associated with rockfall hazards following a Bayesian approach is provided in Chapter 5.

# 2 KNOWLEDGE AND UNCERTAINTY

Knowledge about the considered decision context is a main success factor for optimal decision making. In real world decision making, uncertainty or lack of knowledge however characterises the normal situation and it is thus necessary to be able to represent and deal with this uncertainty in a consistent manner. In the context of societal decision making with time horizons reaching well beyond individual projects or the duration of individual decision makers, the uncertainty related to system assumptions are of tremendous importance. For example, different assumptions can be postulated in regard to future climatic changes, economic developments, long term effects of pollution etc. If the wrong assumptions are made, it is obvious that wrong decisions may be reached. It is thus important to account for uncertainties and their dependencies in the estimation of risks, commensurate to a degree that encapsulates the influence of the uncertainties and their dependencies on the assessed risks.

The consistent treatment of knowledge and the associated uncertainty play a key role not least when managing risks for portfolios of assets; the consistent representation of knowledge and uncertainty assures that results of risk estimates obtained for different assets and for individual hazards can be integrated and aggregated (JCSS 2008). This implies that risk assessment for  e.g. objects or networks subject to several different types of hazards such as e.g. traffic accidents as well as rockfall can be performed by integrating the different model components from the corresponding application areas with due consideration to the uncertainties which influence these models. In the same way risks assessed for two different objects may also be aggregated. If different system representations could be valid, due to lack of knowledge, it is essential to take this into account in the process of risk-based decision making. This is e.g. the case when considering possible future climatic changes, when modelling extreme earthquake excitation and when assessing consequences for hazard events which go beyond historically recorded experience.

There exist a large number of propositions for the characterization of different types of uncertainties. It has become standard to differentiate between uncertainties due to inherent natural variability, model uncertainties and statistical uncertainties. Whereas the first mentioned type of uncertainty is often denoted aleatory (or Type 1) uncertainty, the two latter are referred to as epistemic (or Type 2) uncertainties. Without further discussion here it is just stated that, in principle, all prevailing types of uncertainties can be taken into account in engineering decision analysis within the framework of Bayesian probability theory; a more detailed treatment of this issue is given in Paté-Cornell (1996) and Lindley (1976).

Within such a framework, it is useful to distinguish between the aleatory and epistemic uncertainties. This distinction has been considered for risk assessment of technical systems, e.g., Apostolakis (1990) or Helton and Burmaster (1996), and increasingly for natural hazards, e.g., Hall (2003), Apel et al. (2004) or Straub and Der Kiureghian (2008), but has been discussed also for general geological applications by Mann (1993). Aleatory uncertainties are interpreted as random uncertainties, which, for a given model, are naturally inherent to the considered process; epistemic uncertainties are related to our incomplete knowledge of the process, often because of limited data and can be characterised in the form of model uncertainties and statistical uncertainties.

The absolute and relative magnitudes of aleatory and epistemic uncertainty are markedly case-specific. The differentiation into aleatory uncertainties and epistemic uncertainties is subject to a defined model of the considered system. The relative contribution of the two components of uncertainty depends on the spatial and temporal scale applied in the model. McGuire et al. (2005) argue that for those concerned with application of probability in a decision-theoretic perspective, the differentiation of uncertainty into alreatory and epistemic has no practical consequence. In this case, probabilities are deemed to reflect, in effect, the personal probability values that a decision maker is prepared to act upon. As a simple example, if one's decision about how to bet on a particular poker hand is the same before the cards are shuffled (pure aleatory variability) as they are after five cards are dealt face down on the table (pure epistemic uncertainty), then our equality of preference between the two cases implies that we are assigning them the same probability and acting as if there is no difference between aleatory and epistemic uncertainties. In that case, the probability of an event (that has been defined in turn by probabilities and conditional probabilities of events segregated for whatever practical, operational reasons into epistemic and aleatory) is the average aleatory probability. The averaging, again, is over the epistemic probabilities. The formal basis for this interpretation lies in decision theory (e.g., Savage 1954, Raiffa 1968) which has the use of probability firmly in mind.

The differentiation in uncertainties is introduced for the purpose of setting focus on how uncertainty may be reduced, rather than calling for a differentiated treatment in the risk assessment and decision analysis process. The distinction is relevant because aleatory uncertainty cannot be reduced for a given model. In contrast, epistemic uncertainty can be reduced, for instance, by collecting additional information. For this reason, a clear identification of the epistemic uncertainties in the analysis is crucial, as these may be reduced at a later time. Furthermore, neglecting epistemic uncertainty can lead to strong underestimation of the risk, see Coles et al. (2003) for an example.

# 3 UNCERTAINTIES IN MODELS AND PARAMETERS USED IN RISK ASSESSMENT AND MANAGEMENT

## 3.1 INTRODUCTION

The process of risk assessment involves the quantification and estimation of risk which, in turn, requires the use of models to describe the probability of occurrence of considered events and also the consequences of these events. Models that are used include those for triggering events, process of the landslide/rockfall (including soil and rock slope properties), damages to infrastructure, injuries and loss of lives, and damages to environment and follow-up consequences and socio-economic impact. This chapter provides guiding principles and as basis for the modelling and representation of the underlying uncertainties in the use of these models for the quantification and estimation of risks.

## 3.2 GUIDING PRINCIPLES

It is important that any risk assessment exercise should include a description of all relevant assumptions made in connection with the identification of the system for analysis and its components, as well as the modelling of the associated consequences and frequencies. The level and type of knowledge available to support the assumptions, as well as the modelling of consequences and frequencies, should be explicitly stated. In some cases information is available in terms of observations of e.g. accidents or events of natural hazards. In case such information is available, it should always be attempted to utilize this for the modelling of frequencies of the events. The same applies for consequences for which experience from previous events might be utilized. Such models should account for the statistical uncertainty representing the effect of limited data as well as possible model uncertainties originating from the use of the models for other cases than the case from which they were obtained. In many cases parameters which are known to have influence on the risks are simply not known in a given situation. This may e.g. be the case if the aggregated risk for all tunnels is assessed without accounting for detailed information about the geometry of the tunnels. In such cases it is important to account for the lack of knowledge, by representing the unknown tunnel geometry parameters as uncertain parameters in the formulation of the risk analysis models.

Commonly the assessment of frequencies and consequences depend on models based on experience and engineering understanding. In such cases the uncertainty associated with the models should be described preferably in quantitative terms. In general the documentation of the knowledge should address all relevant uncertainties due to inherent natural variability as well as model uncertainties and statistical uncertainties. Independent of whether such uncertainties are neglected, assessed qualitatively or quantitatively, their treatment and modelling should always be stated clearly as a general rule. Neglecting uncertainties in the risk assessment should always be justified by sensitivity studies.

A rational and solid basis for the quantitative representation of uncertainties in risk assessment can be derived from the theory of probability. Bayesian statistics provides a basis for the consistent representation of uncertainties independent of their source and readily facilitates the joint consideration of purely subjectively assessed uncertainties, analytically assessed uncertainties and evidence as obtained through observations. All uncertainties should be represented in accordance with available data and/or unbiased estimated, based on experience and expertise. It is underlined that possible extreme consequences may be subject to considerable uncertainty, due to the fact that only very little information and experience is available on these. Indirect consequences due to the perception of adverse events by the public are poorly understood and the associated large uncertainty should be accounted for accordingly in the risk assessment.

Generally, uncertainties are best represented through random variables with specified probability density functions and corresponding parameters. If two or more uncertainties can be assumed to be statistically or otherwise dependent, this dependency should be accounted for in the probabilistic modelling. Statistical dependency may be appropriately represented through correlation. Functional dependency or common cause dependency is appropriately represented through hierarchical probabilistic models. Only if the prevailing dependencies are correctly accounted for when assessing the risks for different objects and systems will it be possible to aggregate the risks correctly.

As an example, consider the risks due to rockfall events along a road. There may be several objects such as tunnels, bridges and galleries along the road, each separated by roadway segments. The aggregated risk for the considered road can conveniently be assessed through the sum of the risks for all objects. However, in this case the risks for each of the objects depend on common factors including the average traffic volume per hour (over the day), the time of the event of rockfall (day/night), the type of traffic and the consequences due to disruption, each of which might be associated with uncertainty. When assessing the total risk aggregated over all objects on the considered roadway, it is thus necessary to aggregate the risks conditional on the common uncertain parameters first and thereafter to integrate the aggregated risks over the uncertain common parameters. In principle this operation is quite simple, but in essence very important, as it will yield a significant effect on the aggregated risks. If risks are aggregated without consideration of common uncertain factors, the resulting total risk may be grossly at error.

## 3.3 BASIS OF UNCERTAINTY MODELLING

### 3.3.1 Basic variables

Any model can be considered to contain a specified set of basic variables, i.e. physical quantities which characterize actions and environmental influences, material and soil properties and geometrical quantities. The model may also contain model parameters which characterize the model itself and which are treated as basic variables. Finally there are also parameters which describe the requirements (e.g. serviceability constraints) and which may be treated as basic variables. The basic variables (in the wide sense given above) are assumed to carry the entire input information to the calculation model.

The basic variables may be random variables (including the special case deterministic variables) or stochastic processes or random fields. Each basic variable is defined by a number of parameters such as mean, standard deviation, parameters determining the correlation structure etc.

### 3.3.2 Types of uncertainty

Uncertainties from all essential sources must be evaluated and integrated in a basic variable model. Types of uncertainty to be taken into account are:

- intrinsic physical or mechanical uncertainty
- statistical uncertainty, when the design decisions are based on a small sample of observations or when there are other similar conditions
- model uncertainties.

The types of probability distributions of the basic variables can be standardised, usually within different classes of decision problems. For example, such standardisations for structural design problems can be found in the Probabilistic Model Code of the Joint Committee on Structural Safety (JCSS 2001).

### 3.3.3 Definition of populations

The random quantities used within models for a risk assessment process need to be related to a meaningful and consistent set of populations. The description of the random quantities should correspond to this set and the resulting failure probability is only valid for the same set. The basis for the definition of a population is in most cases the physical background of the variable. Factors which may define the population are:

- the nature and origin of a random quantity
- the spatial conditions (e.g. the geographical region considered)
- the temporal conditions (e.g. the intended time of use of the structure considered)

The choice of a population is to some extent a free choice of the modeller. It may depend on the objective of the analysis, the amount and nature of the available data and the amount of work that can be afforded. In connection with theoretical treatment of data and with the evaluation of observations, it is often convenient to divide the largest population into sub-populations which in turn are further divided in smaller sub-populations etc. Then it is possible to study and distinguish variability within a population and variability between different populations. In an analysis for a specific structure, it may be efficient to define a population as small as possible as far as use, shape and location of the structure are concerned (this is referred to as micro-zonation). When the results are used for design on a larger scale (for example in a national or international code), it may be necessary or convenient to put the sub-populations together to the large population again in order not to get too complicated rules, meaning that the variability within the population is now increased.

### 3.3.4 Hierarchy of uncertainty models and scales of modelling variations

This section contains a convenient and recommended mathematical description in general terms of a hierarchical model which can be used for different kinds of hazards as well as materials. The details of this model have to be stated more precisely for each specific variable. The model is associated with a hierarchical set of subpopulations. The hierarchical model assumes that a random quantity X can be written as a function of several variables, each one representing a specific type of variability:

$$X_{ijk} = f\,(Y_i, Y_{ij}, Y_{ijk}) \qquad (3.1)$$

The quantities $Y_i$, $Y_{ij}$ or $Y_{ijk}$ represent various origins, time scales of fluctuation or spatial scales of fluctuation. For instance $Y_i$ may represent the building to building variation, $Y_{ij}$ the floor to floor variation in building i and $Y_{ijk}$ the point to point variation on floor j in building i. In a similar way, $Y_i$ may represent the constant in time variability, $Y_{ij}$ a slowly fluctuating time process and $Y_{ijk}$ a fast fluctuating time process.

It is often useful to distinguish between three hierarchical levels of variation: macro (global), meso (local) and micro, particularly as far as the spatial variations are concerned,. For example, the variability of the mean and standard deviation of a material property such as the compressive strength of concrete expressed as cylinder strength per job or construction unit is a typical form of global parameter variation. This variation primarily is the result of production technology and production strategy of the concrete producers. Such parameter variations between objects are conveniently denoted as macro-scale variations. The unit of that scale is in the order of a structure or a construction unit. Parameter variations may also be due to statistical uncertainties.

Given a certain parameter realisation in a system, the next step is to model the local variations within the system in terms of random processes or fields. Characteristically, spatial correlations (dependencies) become negligible at distances comparable to the size of the system. This is a direct consequence of the hierarchical modelling procedure where it is natural to assume that the variation within the system is conditional on the variations between systems and the first type of variation is conditionally independent of the second. At this level it is useful to speak in terms of meso-scale variations. Examples are the spatial variation of soils within a given (not too large) foundation site or the number, size and spatial distribution of flaws along welding lines given a welding factory (or welding operator). The unit of this scale is in the order of the size of the structural elements and probably most conveniently measured in metres.

At the third level, the micro-level, the focus is on rapidly fluctuating variations and non-homogeneities which basically are uncontrollable as they originate from physical facts such as the random distribution of spacing and size of aggregates, pores or particles in concrete, metals or other materials. The scale of these variations is measured in particle sizes, i.e. in centimetres down to the size of crystals.

### 3.3.5 Model uncertainties

A calculation model is a physically based or empirical relation between relevant variables, which are in general random variables:

$$Y = f\,(X_1, X_2, \ldots X_n) \tag{3.2}$$

where Y is the model output, f ( )is the model function and $X_i$ are the basic variables.

The model f (...) may be complete and exact, so that, if the values of $X_i$ are known in a particular experiment (from measurements), the outcome Y can be predicted without error. This, however, is not normally the situation. In most cases the model will be incomplete and inexact. This may be the result of lack of knowledge, or a deliberate simplification of the model, for the convenience of the modeller. The difference between the model prediction and the real outcome of the experiment can be expressed as:

$$Y = f'\,(X_1 \ldots X_n, \theta_1 \ldots \theta_m) \tag{3.3}$$

$\theta_i$ are referred to as parameters which contain the model uncertainties and are treated as random variables. Their statistical properties can in most cases be derived from experiments or observations. The mean of these parameters should be determined in such a way that, on average, the calculation model correctly predicts the test results.

## 3.4 EXAMPLE – UNCERTAINTIES IN SOIL PROPERTIES

### 3.4.1 Introduction

In this section, a short discussion on uncertainties pertaining to soil properties taken from JCSS (2001) is provided. Here, the term “soil properties” refers to a collection of characteristics of a soil body, which affect the response to loading or other actions. These include:

- soil stratigraphy, i.e. boundaries for sub-volumes containing a single soil type (denoted as a soil unit)
- continuum properties, such as physical or mechanical parameters or state properties within each of the soil units, e.g. stiffness, compressibility, shear strength, permeability, over-consolidation ratio, initial pore pressures, etc.

Together with basic characteristics of behaviour of each soil unit, e.g. drained, undrained or partially drained behaviour, these items constitute the basic components of a soil model. Distinction of soil units is usually made on the basis of lithological and geotechnical classification (sand, clay, organic material, or mixtures, compaction and consistency) and basic characteristics of response to loading. Although reliability of foundations or other structures in soils is also controlled by uncertainties of applied loads and other construction materials (concrete or steel), a characteristic feature of geotechnical structures is the dominating role of the uncertainties of soil properties. Different sources of uncertainty of soil properties may be distinguished:

- spatial variability of soil properties. Patterns of variability may be either continuous or discrete
- limited soil survey and laboratory or in situ testing
- inaccuracy of soil investigation methods and erroneous interpretation of investigation results.

### 3.4.2 Continuous spatial variability

Continuum properties of a soil unit may vary continuously from one spot to another throughout the unit. The pattern of variability may be characterized by an average trend of variation, e.g. increase with depth, and continuous fluctuations around the average trend. This type of characterization also applies to continuously varying boundaries of soil units, e.g. depth level and thickness of a soil layer. Usually, this type of variability is modelled as a continuous stationary random field; further details can be found in the Probabilistic Model Code of the Joint Committee on Structural Safety (JCSS 2001).

Parameters in a geotechnical analysis usually refer to averages of continuum properties over some surface area or some volume; e.g. average shear strength along a sliding surface or average stiffness of a volume affected by loading. Hence, relevant uncertainties of soil parameters in a geotechnical analysis concern usually uncertainties of its averages over affected surfaces of volumes. Random field modelling of "point to point" variations forms the basis for quantitative assessment of uncertainties of averaged soil parameters.

### 3.4.3 Discrete spatial variability

Soil units with continuous spatial variability may be mixed with dislocations such as faults, lenses or fills, depending on the geological and morphological history. Though local of nature, these phenomena may have a large effect on behaviour of structures built on or in the ground. Often exact locations and sizes of local phenomena are difficult to infer from, if at all revealed by, soil survey campaigns.

### 3.4.4 Limited soil survey and testing

Information about subsurface conditions is acquired by field investigation in discrete survey points (tested samples from borings, SPT-records) or at discrete survey lines (CPT or geophysical records). Soil data is therefore generally only available for a small part of the relevant soil volume which implies, as a consequence, uncertainties which are somehow statistical of nature. Two types may be distinguished:

- inaccurate statistics of soil property distributions (continuum parameters and continuous soil layer boundaries)
- potential errors in the soil stratigraphy (e.g. missing local phenomena, anomalies).

Both types of uncertainties may be reduced at the expense of additional survey or testing. Considering continuous soil properties, the effect of additional survey and testing is the reduction of the error of estimated statistics by means of statistical sample theory or geo-statistical approaches.

Considering potential errors of soil stratigraphy, the effect of additional survey is likely to be a reduction of the probability of occurrence of such errors. However, the process of assessment of soil stratigraphy from available soil data is for a significant part based on subjective engineering judgement. Hence, quantification of probabilities of stratigraphical errors, and its reduction due to additional survey, is also subject of engineering judgement. Probabilistic approaches to assess probabilities of occurrence of potential errors in the soil stratigraphy, related to type, extent and intensity of soil investigation, are far from well developed. Yet it seems that the effects of such errors can be more drastic than the effects of inaccurate statistics of continuous soil properties.

### 3.4.5 Inaccuracy of soil investigation method

Inaccuracies may be caused by sample disturbance, test imperfections, such as poor reproducibility of tests or poor correlation between in situ test results and basic soil parameters, and human factors in conducting tests and interpretation of soil investigation results. Though this type of inaccuracies is often not the least important source of uncertainty, only part of it can be taken into account explicitly in probabilistic analyses. Gross errors of test equipment and in conducting tests must be avoided by appropriate quality assurance procedures. Gross errors in interpretation of soil investigation must be avoided by a thorough control scheme and an expert review.

# 4 A BAYESIAN APPROACH TO DEAL WITH UNCERTAINTY

## 4.1 INTRODUCTION

The treatment of uncertainty can be closely related to the interpretation of probability. In this regard, a distinction into three possible interpretations can be made (JCSS 2001):

- the frequentistic interpretation
- the formal interpretation
- the Bayesian interpretation

The frequentistic interpretation is quite straightforward and allows only “observable and countable” events to enter the domain of probability theory. Probabilities need to be based on a sufficient number of data or on unambiguous theoretical arguments only (as in coin flipping or die throwing games). Such an interpretation, however, can only be justified in a stationary world where the amount of statistical or theoretical evidence is very large. It can be said that such an interpretation is out of the question in most areas of risk assessment and decision making. In almost all cases, the data is too scarce and often only of a very generic nature.

The formal interpretation gives full credit to the fact that the numbers used in a reliability and risk analysis are based on ideas and judgment rather than statistical evidence. Probabilistic structural design, for example, is considered as a strictly formal procedure without any physical interpretation. Such a procedure, nevertheless, is believed to be a richer and more consistent design procedure compared to classical and deterministic structural design methods. However, in many cases it is convenient if the values in the probabilistic calculations have some meaning and interpretation in the real world. One relevant example is that one should be able to improve or update the probabilities in the light of new statistical evidence or new knowledge.

This leads into the direction of a Bayesian probability interpretation, where probabilities are considered as the best possible expression of the degree of belief in the occurrence of a certain event. The Bayesian interpretation does not claim that probabilities are direct and unbiased predictors of occurrence frequencies that can be observed in practice. The only claim is that the probabilities will be more or less correct if averaged over a large number of decision situations. The requirement to fulfill that claim, is that the purely intuitive part is neither systematically too optimistic nor systematically too pessimistic. Calibration to common practice on the average may be considered as an adequate means to achieve that goal.

In simplistic terms – in the Bayesian approach, the aleatory or inherent uncertainties are treated in the frequentistic way and the epistemic or knowledge uncertainties are treated in a degree of belief way. The basic advantage of the Bayesian approach above the other approaches is that the "degree of belief" becomes exactly equal to a "frequentistic probability" in the limiting case of strong evidence like huge statistics or closed theoretical arguments. This property ensures a clear interpretation of the calculations and enables the combination of several sources of evidence. Another advantage is that one has the fully developed and strong theory of probability at ones disposition for both types of uncertainty. Further, a consistent consideration of new knowledge or information as and when it becomes available is possible in the Bayesian approach through updating.

## 4.2 BAYESIAN PROBABILISTIC MODELLING

Consistent decision making subject to uncertainties is treated in detail in Raiffa and Schlaifer (1961) and Benjamin and Cornell (1970). Other aspects on decision analysis in engineering applications are treated in Apostolakis (1990), Paté-Cornell (1996) and Faber and Stewart (2003). In this section, an introduction to three different decision analyses is given – namely prior, posterior and pre-posterior decision analysis. Risk assessment and risk management are integral to the decision analysis process and hence a clear consideration and treatment of risk and the underlying uncertainties is also provided.

### 4.2.1 Prior decision analysis

The simplest form of the decision analysis is prior analysis. In the prior analysis, the risk (expected utility) is evaluated on the basis of statistical information and probabilistic modelling available prior to any decision and/or activity. This prior decision analysis is illustrated by a simple decision tree in Figure 4.1. In prior decision analysis the risk (expected utility) for each possible decision activity/option is evaluated in the principal form as:

$$R = E[U] = \sum_{i=1}^{n} P_i \, C_i \tag{4.1}$$

where *R* is the risk, *U* the utility, $P_i$ is the $i^{th}$ branching probability (the probability of state *i*) and $C_i$ the consequence of the event of branch *i*.

Prior decision analysis in fact corresponds closely to the assessment of the risk associated with an activity. Prior decision analysis thus forms the basis for the simple comparison of risks associated with different activities. The result of a prior decision analysis might be that the risks are not acceptable and the risk reducing measures need to be considered.
In structural engineering a typical prior decision analysis is the design problem. A design has to be identified which complies with given requirements to the structural reliability. The representation of uncertainties is made on the basis of the existing information about materials and loads, however, as these have not occurred yet the probabilistic modelling involve both aleatory and epistemic uncertainties. As a general comment it should be noted that in the context of setting requirements to reliability and risk it is necessary to ensure consistency between the probabilistic models used for setting the requirements and the probabilistic models used for their verification.

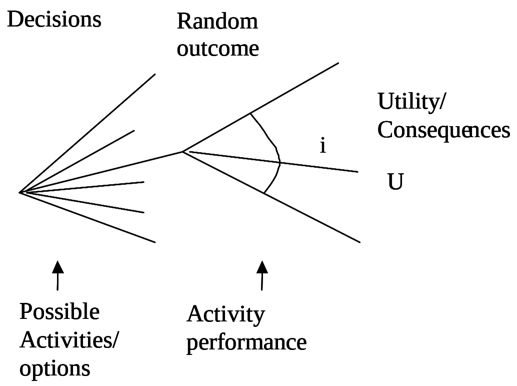


*Figure 4.1 Decision tree for prior and posterior decision analysis*

### 4.2.2 Posterior decision analysis

Posterior decision analysis is in principle of the same form as the prior decision analysis, however, changes in the branching probabilities and/or the consequences in the decision tree reflect that the considered problem has been changed as an effect of e.g. risk reducing measures, risk mitigating measures and/or collection of additional information. Posterior decision analysis may thus be used to evaluate the efficiency of risk reducing activities with known performances. The posterior decision analysis is maybe the most important in engineering applications as it provides a means for the utilization of new information in the decision analysis – referred to as updating; this is described in short in the following.

#### *4.2.2.1 Uncertainty updating – updating of random variables*

Inspection or test results relating directly to realizations of random variables may be used in the updating. The distribution parameters are initially (and prior to any update) modeled by prior distribution functions.

By application of Bayes theorem, see e.g. Lindley (1976), the prior distribution functions, assessed by any mixture of frequentistic and subjective information, are updated and transformed into posterior distribution functions.

Assume that a random variable $X$ has the probability distribution function $F_X(x)$ and density function $f_X(x)$. Furthermore assume that one or more of the distribution parameters, e.g. the mean value and standard deviation of $X$ are uncertain themselves with probability density function $f_Q(q)$. Then the probability distribution function for $Q$ may be updated on the basis of observations of $X$, i.e. $\hat{x}$.

The general scheme for the updating is:

$$f_Q^{''}(q \mid \hat{x}) = \frac{f_Q^{'}(q)\, L(q \mid \hat{x})}{\int_{-\infty}^{\infty} f_Q^{'}(q)\, L(q \mid \hat{x})\, dq} \tag{4.2}$$

where $f_Q(q)$ is the distribution function for the uncertain parameters $Q$ and $L(q|\hat{x})$ is the likelihood of the observations or the test results contained in $\hat{x}$. Here ´´ denotes the posterior and ´ the prior probability density functions of *Q*. The observations $\hat{x}$ may not only be used to update the distribution of the uncertain parameters *Q*, but also to update the probability distribution of *X*. The updated probability distribution function for *X* $f_X^U(x)$ is often called the predictive distribution or the Bayes distribution. The predictive distribution may be assessed through

$$f_X^U(x) = \int_{-\infty}^{\infty} f_X(x|q)\, f_Q^{''}(q \mid \hat{x})\, dq \tag{4.3}$$

In Raiffa and Schlaifer (1961) and Aitchison and Dunsmore (1975) a number of closed form solutions to the posterior and the predictive distributions can be found (also collected in JCSS 2000) for special types of probability distribution functions known as the natural conjugate distributions. These solutions are useful in updating of random variables and cover a number of distribution types of importance for reliability based structural reassessment. However, in practical situations there will always be cases where no analytical solution is available. In these cases FORM/SORM techniques (Madsen et al. 1986) may be used to integrate over the possible outcomes of the uncertain distribution parameters and in this way allow for assessing the predictive distribution.

#### *4.2.2.2 Probability updating - updating of uncertain relations*

In many practical problems the observations made of realizations of uncertain phenomena cannot be directly related to random variables. In such cases other approaches must be followed to utilize the available information. Given an inspection result or other observation of an outcome of a functional relationship between several basic variables, probabilities may be updated using the definition of conditional probability or its extension known as Bayes formula:

$$P(F|I) = \frac{P(F \cap I)}{P(I)} = \frac{P(I|F)P(F)}{P(I)} \tag{4.4}$$

*F* = Failure
*I* = Inspection result

For a further evaluation of Equation (4.4) it is important to distinguish between the types of inspection results. For inequality type information which may be expressed by limit states of the following form $h(\mathbf{X}) < 0$, Equation (4.4) may be elaborated in a straightforward way. Let *F* be represented by $M(\text{x}) \leq 0$, where *M* denotes the safety margin. We then have:

$$P(F|I) = \frac{P(M(\mathbf{X}) \leq 0 \cap h(\mathbf{X}) < 0)}{P(h(\mathbf{X}) < 0)} \tag{4.5}$$

where X = vector of random variables having the prior distribution $f_{\mathbf{X}}(\mathbf{x})$. This procedure can easily be extended to complex failure modes and to a set of inspection results ($\cap h_i(\text{x}) < 0$).

### 4.2.3 Pre-posterior decision analysis

Using pre-posterior decision analysis optimal decisions in regard to information collection activities, which may be performed in the future, can be identified. Pre-posterior decision analysis is excellently described in e.g. Raiffa and Schlaifer (1961) and Benjamin and Cornell (1970). The principle behind the pre-posterior decision analysis is that the outcomes of planned information collection activities are assumed to follow the prior probabilistic model of uncertainties. Based on these assumed outcomes and taking into account any uncertainties associated with the observation and/or interpretation of the outcomes posterior decision analyses are performed. The corresponding risks are thereafter weighed with their probability of occurrence, again based on the prior probabilistic modelling. The pre-posterior may thus be interpreted as a posterior decision analysis made before new information is actually collected. The principle is also illustrated by the decision tree shown in Figure 4.2. An important pre-requisite for pre-posterior decision analysis is that decision rules specifying future actions which will be taken on the basis of the results of the planned information collection activities need to be formulated.

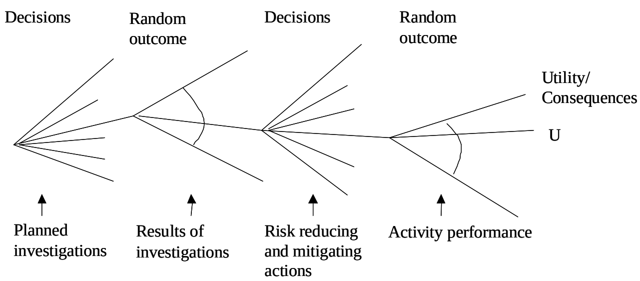


*Figure 4.2 Decision tree for pre-posterior decision analysis*

In pre-posterior decision analysis, the risk (expected utility) for each of the possible risk reducing activities is evaluated as

$$\begin{aligned} R &= E[U] \\ &= \min_a E'_{\mathbf{Z}}\left[E''_{|\mathbf{Z}}\left[U(a(\mathbf{z}),\mathbf{z})\right]\right] \\ &= \min_a E'_{\mathbf{Z}}\left[\sum_{i=1}^{n} P_i\,''(a(\mathbf{z}),\mathbf{z})C_i(a(\mathbf{z}))\right] \end{aligned} \tag{4.6}$$

where $a(\mathbf{z})$ are the decision rules describing the different possible actions that can be take on the basis of the result of the considered investigation $\mathbf{z}$, $E[\ ]$ is the expected value operator. $P_i(\ )C_i(\ )$ is the product of the probability of the *i*'th event resulting from the decision and the corresponding consequences. ´ and ´´ refer to the probabilistic description of the events of relevance based on prior and posterior information respectively, see e.g. Lindley (1976).
Pre-posterior decision analysis forms a strong decision support tool and has been intensively used for the purpose of risk based inspection planning see e.g. Faber (2002). However, so far pre-posterior decision analysis has been grossly overlooked in risk assessments in general.

### 4.2.4 Uncertainty representation in updating

As mentioned earlier, it is important to differentiate between the different types of uncertainty in the probabilistic modelling of uncertain phenomena. Only when the origin and the nature of the prevailing uncertainties are fully understood a consistent probabilistic modelling can be established allowing for rational decision making regarding risk reduction by means of posterior and pre-posterior decision analysis. In the following the representation of uncertainties for representative posterior and pre-posterior decision problems is thus addressed and discussed.

#### *4.2.4.1 Uncertainty modelling in posterior decision problems*

In engineering decision analysis posterior decision problems typically involve the updating of the probability of a future adverse event $F$, $P_F^U$ conditional on the observation of an event $I$ which can be related to the adverse event. Such observations may in general be considered as being indications about the adverse event. The probability $P_F^U$ may be assessed by Equation (4.4):

$$P_F^U = P(F|I) = \frac{P(F \cap I)}{P(I)} \tag{4.7}$$

Taking basis in Equation (4.7) a simple case is now considered where the adverse event is a future ($\tau \in [t,T]$) failure event in terms of a load $S(\tau)$ exceeding the resistance *R* of an existing structural component. Furthermore it is assumed that the indicator *I* is the event that the component has survived all previous realizations of the loading $S(\tau)$ $\tau \in [0,t]$. Then Equation (4.7) can be written as:

$$P_f^U(T) = P(F|I) = \frac{P\left(\left\{\min_{\tau\in[t,T]}(R - S(\tau)) \le 0\right\} \cap \left\{\min_{\tau\in[0,t]}(R - S(\tau)) > 0\right\}\right)}{P\left(\left\{\min_{\tau\in[0,t]}(R - S(\tau)) > 0\right\}\right)} \quad (4.8)$$

In accordance with the considerations made in the previous $R$ is an epistemic uncertainty since it has already had its realization but it is still unknown and thus uncertain. As long as no other information is available it would be consistent to model the epistemic uncertainty associated with $R$ using the same model assumptions (distribution type etc.) as before $R$ had its realization. $S(\tau)$ is "in principle" an aleatory uncertainty (assuming that no model and/or statistical uncertainties are involved in the modelling of the load) when we consider future loads i.e. for $\tau \in [t,T]$. The uncertainty associated with $S(\tau)$ is of an epistemic nature when we consider already occurred load events, i.e. $\tau \in [0,t]$. The wording "in principle" is used because the temporal dependency characteristics of the loading $S(\tau)$ play a significant role. If the load events (or extreme loads) in consecutive time intervals are assumed to be conditional independent – a relatively normal case in engineering problems – then the consideration outlined in the above are valid. This also implies that the uncertainty associated with the future loading cannot be updated on the basis of observations of the past loading. However, if the load events in consecutive time intervals are dependent then a part of the uncertainty associated with the future loading becomes epistemic as soon as its first realization has occurred. The "size" of the part depends on the temporal dependency.

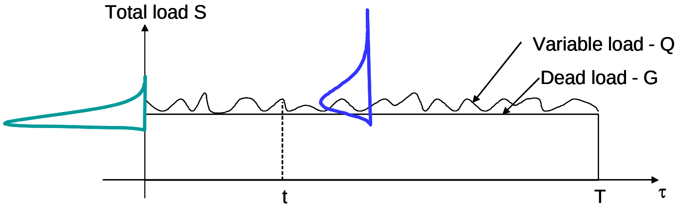


*Figure 4.3 Illustration of a load with high degree of temporal dependency*

In Figure 4.3, it is illustrated how the load events in consecutive time intervals may be highly dependent due to e.g. a dominating dead load component. Before the dead load component is realized the loading in the future might be subject to aleatory uncertainty only. As soon as the dead load component is realized a large part of the uncertainty associated with the future loading becomes epistemic. This effectively implies that this part of the uncertainty associated with the future loading can be updated on the basis of observations of the past loading. In other words – the part that can be updated is exactly the epistemic part of the uncertainty. If the probabilistic modelling of the uncertainties and the probability updating is performed in accordance with Equation (4.8) and the considerations outlined in the above then, the resulting probabilistic modelling and the representation of the different types of uncertainties is consistent. However, if in the representation of the adverse event and the updating event the different types of uncertainty and the temporal dependency is not consistently taken into account, the results may become grossly erroneous and non-physical.

#### *4.2.4.2 Uncertainty modelling in pre-posterior decision problems*

As can be realized from Equation (4.6), pre-posterior decision problems may be seen as a series of posterior decision problems for which the optimal solutions are averaged out over the entire prior uncertainty. The formulation of each of the posterior decision problems is based on an updated probabilistic model of the prevailing uncertainties assuming a given "outcome of nature". Therefore the considerations made for posterior decision analysis, concerning the treatment of uncertainties are also valid for pre-posterior decision problems.

### 4.2.5 Use of Bayesian probabilistic networks (BPN)

The risk assessment and management of natural hazards such as landslide and rockfall events requires a systematic and consistent representation and management of information for a typically complex system with a large number of constituents or sub-systems. Such representation must enable a rational treatment and quantification of the various uncertainties discussed earlier; these uncertainties can be associated with the constituents as well as the system. The consistent handling of new knowledge about the system and its constituents as and when it becomes available and its use in the risk assessment and decision making process is also essential. Further, the numerous dependencies and linkages that exist between different constituents of the system need to be systematically considered. The above requirements and considerations necessitate the use of generic risk models for the assessment and management of risks due to natural hazards. The use of Bayesian Probabilistic Networks (BPNs) has proven to be efficient in such risk assessment applications (Graf et al., 2009; Faber et al., 2007; Nishijima and Faber, 2007; Bayraktarli et al., 2006; Bayraktarli et al., 2005; Faber et al. 2005; Schubert et al, 2005 and Straub, 2005). A brief overview of the principles and use of Bayesian Probabilistic Networks is provided below; details can be found in Jensen (2001).

Formally, Bayesian probabilistic networks (BPN) are directed acyclic graphs whose nodes represent random variables in the Bayesian sense: they may be observable quantities, latent variables, unknown parameters or hypotheses. Edges represent conditional dependencies; nodes which are not connected represent variables which are conditionally independent of each other. Each node is associated with a probability function that takes as input a particular set of values for the node's parent variables and gives the probability of the variable represented by the node. Efficient algorithms exist that perform inference and learning in BPNs. Using a BPN offers many advantages over traditional methods of determining causal relationships. Independence among variables is easy to recognize and isolate while conditional relationships are clearly delimited by a directed graph edge: two variables are independent if all the paths between them are blocked (given the edges are directional).

# 5 EXAMPLE – MODELLING OF ROCKFALL HAZARDS

## 5.1 INTRODUCTION

An example taken from Straub and Schubert (2008) concerning the modelling of rockfall hazards and design of rockfall protection structures following a Bayesian approach is considered here. This example has also been reported in Deliverable D0.3 of the SafeLand project (SafeLand 2011).

Rockfall is generally considered an inherently uncertain process, i.e., it is not possible to deterministically predict the time and the extent of the next event. However, it is possible to describe rockfall using a probabilistic model, describing the frequency $H_V(v)$ with which a rock of a certain volume $V$ or larger is detached. Because the assessment of rockfall is based on limited data and simplified models, the probabilistic model is itself subject to uncertainty itself; this can be represented by modelling the parameters of $H_V(v)$ as random variables. In this case, we write $H_V(v|\boldsymbol{\theta})$ to indicate that the model is defined conditional on the values of its parameters $\boldsymbol{\theta}$. This epistemic uncertainty on $\boldsymbol{\theta}$ can be depicted by credible intervals (which can be considered as the Bayesian equivalent of confidence intervals) on the exceedance frequency curve as demonstrated in Figure 5.1.

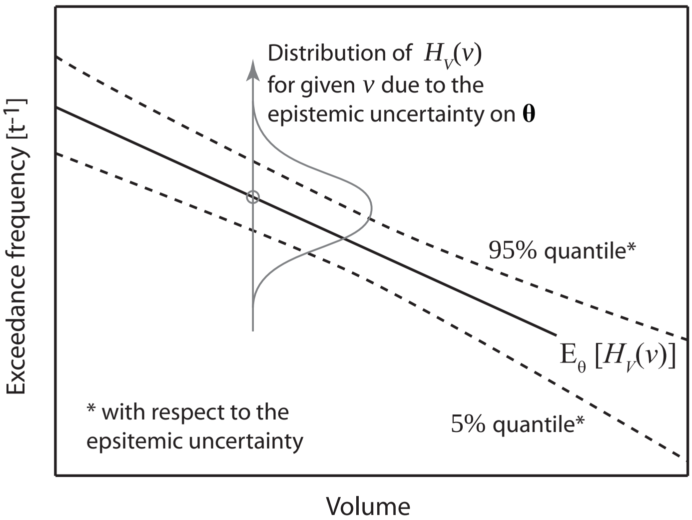


*Figure 5.1 Exceedance frequency – illustrating the difference between epistemic and aleatory uncertainty*

## 5.2 UNCERTAINTIES IN ROCKFALL HAZARDS

As with most natural hazards, the uncertainties related to the occurrence of the hazard are generally large for rockfall hazards. In the literature, this uncertainty is generally represented by an exceedance frequency as illustrated in Figure 5.1, yet without explicit consideration of the epistemic uncertainty. Instead it is (implicitly) assumed that the frequency of an event with a certain rock volume is a deterministic value, implying that if the site were observed

over a sufficiently long period, exactly the predicted frequency of rocks would be experienced. Clearly, this is not the case; instead the predicted frequency is a best estimate of the true rate of occurrence.
In the literature, various methods are proposed for identifying the exceedance frequency at a specific site. These include:

i) the analysis of historical datasets, e.g., Hungr et al. (1999) or Dussauge-Peisser (2002),
ii) empirical models which describe hazard as a function of different indicators (observable parameters) such as topography and geology, e.g., Budetta (2004) or Baillifard et al. (2004),
iii) phenomenological (mechanical) models, e.g., Jimenez-Rodriguez et al. (2006) or Duzgun et al. (2003), and
iv) expert opinion, e.g., Schubert et al. (2005).

All these methods are useful in a particular context. While methods i) and ii) are generally more appropriate for the analysis of larger areas with less accuracy, iii) and iv) are more suited for the detailed analysis of a specific site.

Large-scale models (i) and ii) above) are generally based on statistical methods. Consequently, it is mathematically convenient to express the exceedance frequency in a parametric format. Traditionally, a power law has been applied to describe the relation between rock volume $V$ and the exceedance frequency:

$$H_V\left(v\middle|\boldsymbol{\theta}\right) = av^{-b} \tag{5.1}$$

The statistical parameters of the model characterising the shape of the exceedance frequency curve are $\boldsymbol{\theta} = [a,b]^{\mathrm{T}}$. The epistemic uncertainty is included in the analysis by modelling $\boldsymbol{\theta}$ as a random vector. Using the probability density function of $\boldsymbol{\theta}$, $f_{\boldsymbol{\Theta}}\left(\boldsymbol{\theta}\right)$, the unconditional exceedance frequency is computed as:

$$H_V\left(v\right) = \int_{\boldsymbol{\Theta}} f_{\boldsymbol{\Theta}}\left(\boldsymbol{\theta}\right) H_V\left(v\middle|\boldsymbol{\theta}\right) d\boldsymbol{\theta} \tag{5.2}$$

There are various sources for epistemic uncertainties in large scale models, preventing an exact prediction of the exceedance frequency for a particular site. A brief description of these is provided below.

*Statistical uncertainty*
The parameters of the large scale models are derived empirically from data sets. Because of the limited size of these data sets, the estimated parameters are subject to statistical uncertainty.

*Measurement uncertainty*

Measurements and recordings of the geological properties are typically subject to uncertainty and observations of historical events are often incomplete and biased and must rely on local experts. As an example, rocks on a road will generally be reported and documented, but those that missed the road may often not be. Measurement uncertainty also results from derives from equipment, operator/procedural and random measurement effects.

*Model uncertainty*
Extrapolation of the statistical models to areas other than those for which observations are available leads to additional uncertainty as the geological and topographical characteristics will be different for these areas. GIS-based models will take into account some of these parameters, but the omitted parameters will lead to an uncertainty in the model predictions. Uncertainty also occurs due to the approximations and simplifications inherent in empirical, semi-empirical, experimental or theoretical models used to relate measured quantities to non-measurable numerical parameters used in estimation. Finally, although the power-law is, for example, commonly assumed to express exceedance frequency in the case of rockfall hazard, it has not been justified by phenomenological considerations. Thus, it is not ensured that the parametrical model accurately represents the actual behaviour.

*Spatial variability*
The frequency of hazard events varies in space. The observations represent an average over an area and the resulting parameter values, therefore, do not reflect the variations from the average.

*Temporal variability*
The frequency of hazard events varies in time. When working with annual frequencies, the seasonal changes do not affect the analysis, but the frequency may change over the years or may be dependent on extreme events (e.g., earthquakes). However, in certain instances, e.g., when temporal closure of the road is considered as a risk reduction measure, seasonal variations must be explicitly addressed by the analysis.

How can these uncertainties be quantified? Statistical uncertainty can be quantified by using standard statistical methods such as Bayesian analysis, see, e.g., Coles (2001). Measurement uncertainty can generally be estimated when the data collection method is known. Unfortunately, no simple analytical method is available for estimating model uncertainties. A solution is to rely on expert opinion, i.e., to ask experts about their confidence in the models. It is also possible to compare the model with observations which have not been used in the calibration of the model (model validation) or to compare different models. Furthermore, it is possible to include additional parameters in the formulation of the exceedance frequency. The model uncertainties are then reduced while the statistical uncertainties increase, but the latter can then be estimated analytically. Coles et al. (2003) demonstrate this for the analysis of rainfall data. The spatial and temporal variability can be analysed quantitatively, if data is available in sufficiently small scale; a data-set showing the spatial distribution of rockfall events is presented in Dussauge-Peisser et al. (2002). Spatial variability can be described by the spatial correlation of the relevant characteristics. In most practical cases, however, a simplified approach is favourable, whereby smaller areas are determined within which the spatial variability can be neglected. Temporal (typically seasonal) variability can be described by time-dependent parameters in the exceedance frequency model, corresponding to the assumption of the hazard event (e.g. rockfall) following an inhomogeneous Poisson process. For small-scale models, the application of the power-law is not always justified, in particular if different mechanisms are underlying the detachment of smaller and larger rocks. In such cases it might be more appropriate to utilize a non-parametric model in which the rock volume is divided into a discrete number of intervals (e.g., $10m^3$ – $50m^3$) and the model gives the annual frequency of rocks for the different volume ranges.

## 5.3 BAYESIAN ANALYSIS AND UPDATING

For the modelling of rockfall exposure, Bayesian analysis is particularly useful, as it facilitates the consistent combination of different information in a single model. This is because the probabilistic model can be updated when new information becomes available. Consider the case where rockfall exposure at a particular location is expressed by the model $H_V(v|\boldsymbol{\theta})$ with uncertain parameters $\boldsymbol{\theta}$. When new information becomes available (denoted by $\mathbf{z}$), the probability distribution of the uncertain parameters can be updated using Bayes' theorem, which in its general form can be written as:

$$f_{\Theta}\left(\boldsymbol{\theta}|\mathbf{z}\right) \propto L\left(\boldsymbol{\theta}|\mathbf{z}\right) f_{\Theta}\left(\boldsymbol{\theta}\right) \tag{5.3}$$

$f_{\Theta}\left(\boldsymbol{\theta}\right)$ is the prior probabilistic model, $f_{\Theta}\left(\boldsymbol{\theta}|\mathbf{z}\right)$ is the updated model and $L\left(\boldsymbol{\theta}|\mathbf{z}\right)$ is the likelihood function, which describes the new information. The proportionality constant is obtained from the fact that integration of $f_{\Theta}\left(\boldsymbol{\theta}|\mathbf{z}\right)$ over the entire domain of $\boldsymbol{\theta}$ must yield one. The likelihood function is the probability of the observed information given the parameters $\boldsymbol{\theta}$, i.e.,

$$\mathrm{L}\left(\boldsymbol{\theta}|\mathbf{z}\right) = \Pr\left(\mathbf{z}|\boldsymbol{\theta}\right) \tag{5.4}$$

To demonstrate the derivation of the likelihood function, consider the case where the available information is a set of observed detached rocks $i = 1...n$ for a specific mountain slope, which are described by their volume $v_i$ and the time period $\Delta T_z$ during which they occurred. Only rocks with a volume larger than $v_{th}$ have been recorded (*th*: threshold). We make the following simplifying assumptions: a) that the rockfall follows a homogeneous Poisson process as discussed earlier and b) that the observations are free of error (i.e., all rocks are recorded). These assumptions hold under particular circumstances only, yet they are a reasonable approximation to many real situations and they are suitable for illustrative purposes. Under these assumptions, the probability of observing exactly $n$ rocks with a volume larger than $v_{th}$ is given by the Poisson distribution with parameter $H_V(v_{th}|\boldsymbol{\theta})\Delta T_z$ as:

$$\Pr\left(n|\boldsymbol{\theta}\right) = \frac{\left[H_V\left(v_{th}|\boldsymbol{\theta}\right)\Delta T_z\right]^n}{n!} \exp\left[-H_V\left(v_{th}|\boldsymbol{\theta}\right)\Delta T_z\right] \tag{5.5}$$

Given that a rock with volume larger than $v_{th}$ has detached, the likelihood of its volume being $v_i$ is proportional to $\mathrm{h}_V(v_i|\boldsymbol{\theta})/\mathrm{H}_V(v_{th}|\boldsymbol{\theta})$ for $v_i \le v_{th}$. Because all observations are assumed independent events, the likelihood function is obtained by multiplying these terms. The likelihood function representing the observation of $n$ rocks with volumes $v_1 ... v_n$ on the considered mountain area is then:

$$L(\boldsymbol{\theta}|\mathbf{z}) \propto \exp\left[-H_V\left(v_{th}|\boldsymbol{\theta}\right)\Delta T_z\right]\prod_{i=1}^{n} h_V\left(v_i|\boldsymbol{\theta}\right) \tag{5.6}$$

$h_V(v_i|\boldsymbol{\theta})$ is the annual frequency density of $V$. Note that the observations apparently must relate to the frequency density and not the probability density, because we cannot observe just the largest rock that has fallen during a certain period, rather, the observed rocks may all be from the same time period.

## 5.4 UNCERTAINTIES IN ROCKFALL TRAJECTORY

Once a rock is released, its trajectory is mainly determined by the topography, its mode of motion (free fall, rolling bouncing or sliding) and the characteristics of the surfaces of the rock and the ground. All these factors contribute to the uncertainty in the prediction of the trajectory. Existing numerical tools model this uncertainty by means of crude Monte Carlo simulation (MCS); an overview is provided by Guzzetti et al. (2002). There exist two- or three-dimensional models and there are differences in the physical representation of the rock: The so called lumped mass approach represents the rock by a single mass point, neglecting the geometry of the stone. The rigid body approach models the stone by idealized geometries (e.g., cylinders, spheres or a cuboidal shape, Ettlin 2006) with varying physical and material properties. Hybrid models combine a lumped mass approach to simulate the free fall with a rigid body approach to simulate the contact with the ground surface. Finally, different models are used to simulate the impact of the rock on the ground (Dorren, 2003), a simple approach being the use of coefficients of restitution (Stevens 1998). The impact is the most intricate part of the falling process and its modelling is associated with large uncertainties. The modelling cannot account for the variability in the ground material (particularly in zones covered with vegetation) and the local geometry of the ground and the rock. These uncertainties are inherent to the model and can therefore be considered as aleatory. In addition, there is an epistemic uncertainty because of the limited basis for estimating the model parameters (see e.g., Robotham et al. (1995), Azzoni et al. (1995) and Chau et al. (2002) for estimation of coefficients of restitution). Additional epistemic uncertainty is due to the simplified modelling of the slope profile at the impact location. In many applications, the profile surface in the models is generated from a digital elevation model (DEM) with limited resolution and between the points provided by the DEM the terrain is assumed to be linear. If the model is 2-dimensional, the reduction to a single plane is an additional source of epistemic uncertainty.

The outcome of a two-dimensional rockfall model is illustrated in Figure 5.2. In this example, the relevant numerical result that will be utilized for risk assessment is the probability density function (PDF) of the energy of the rocks when reaching the road. This distribution should be evaluated conditional on the rock volume, $f_E(e|v)$, for different values of $v$. This can then be combined with the distribution representing the rock detachment. Available rockfall analysis software typically allow entering the detached volume as a Normal distributed random variable, but because the volume of detached rocks is generally not Normal distributed, results obtained with this assumption cannot be used for risk assessment directly.

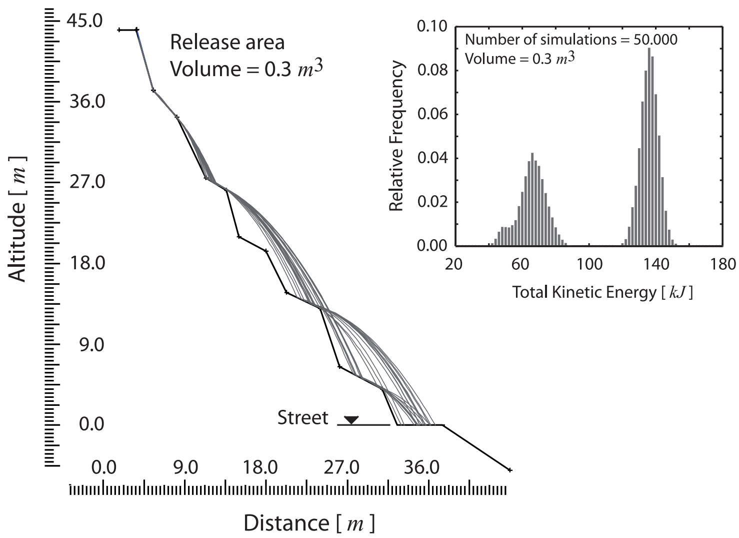


*Figure 5.2 Illustration of the rockfall trajectory modelling.*

MCS in existing rockfall trajectory analysis tools accounts only for the aleatory uncertainty. However, while it is important to be aware of the additional epistemic uncertainty associated with these models, for most practical applications, the error associated with neglecting this uncertainty is tolerable. This is due to the fact that in the analysis of rockfall trajectories, unlike in the modelling of rockfall exposure, the probability of extreme events is of less importance, and that the middle range of the distribution is less affected by the epistemic uncertainties.

## 5.5 UNCERTAINTY IN THE PERFORMANCE OF ROCKFALL PROTECTION STRUCTURES

Rockfall protection structures such as flexible nets or fixed galleries can stop the rocks, but their capacity is limited. This capacity, denoted by *R*, can be quantified in terms of the amount of energy that the structure can absorb. *R* depends on the type of structure, but also on the characteristics of the rock beyond the impact energy. The uncertainty in the capacity is considered by modelling *R* as a random variable, represented by its PDF conditional on the rock volume, $f_R(r|v)$. Hereby, the velocity of the rock at the impact is determined as a function of the energy and the volume. $f_R(r|v)$ should include both epistemic and aleatory uncertainty related to the structural capacity. Structural reliability analysis can be used to evaluate $f_R(r|v)$ for a given type of structure, Schubert et al. (2005). Alternatively, for standard protection systems, $f_R(r|v)$ can also be estimated from tests. However, because of their cost, the number of tests is often limited and, therefore, test results should be combined with a reliability analysis to obtain a probabilistic estimate of the capacity.

## 5.6 UNCERTAINTIES IN ROCKFALL ROBUSTNESS

A measure of how a system such as a rockfall protection structure reacts to a hazard or a damage or failure event can be expressed as the robustness of the system. The robustness of such a system can be accounted for by estimating the expected consequences for a given failure event following the approach described in JCSS (2008). As an example, the expected number of fatalities and injuries is evaluated by multiplying the probability that a number of people are present at the location at the time of a rockfall with the probability that somebody is killed or injured by the rock. Those probabilities represent aleatory uncertainties. There is an uncertainty as to the values of these probabilities, which is of an epistemic nature (it could be reduced by collecting additional data), but because only the expected number of fatalities and injuries enters the computation, the computed risk generally will not be very sensitive to these epistemic uncertainties. In most instances they can be neglected, as is done in practice.

An important part of system robustness modelling is the assessment of so-called “user costs”, representing the socio-economical costs inflicted by the temporary disuse of the considered system, typically a transportation link. The user costs as assessed by road authorities exhibit large differences (e.g. Nash, 2003). However, it must not be concluded that these differences are due to epistemic uncertainty; rather, they are caused by different model assumptions. Therefore, this problem must be addressed by the decision maker, who must determine the model assumptions that represent his/her preferences.

# 6 SUMMARY AND CONCLUSIONS

The consideration and treatment of uncertainties is an essential part of any risk assessment and risk management process. Uncertainties can either be naturally inherent or modelling and statistical related. This deliverable aims to provide a rational basis for the quantification of the various uncertainties existent in the risk assessment and risk management processes. A general description of the uncertainties in the modelling, prediction and decision making processes and their treatment and propagation can be found in Deliverable D0.3 of the SafeLand project. In order to obtain a complete picture of the issues and aspects concerning the treatment, quantification and management of uncertainties in the risk assessment, risk management and decision making processes, it is advised that this deliverable report be read in conjunction with the report of Deliverable D0.3.

The issue of knowledge and uncertainty in the real world decision making platform is introduced in Chapter 2 of this report. Here, a differentiation of uncertainties into aleatory and epistemic uncertainties is introduced, primarily for the purpose of setting focus on how uncertainty may be reduced. In Chapter 3, focus is directed on the different models used for the quantification of risk and the characterisation of parameters in the models. Guiding principles and a general basis for the modelling and representation of the underlying uncertainties in the use of these models for the quantification and estimation of risks are provided. A Bayesian approach is advocated for the representation, handling and management of uncertainties in the context of decision making and is described in Chapter 4. Finally, an example on the modelling and management of uncertainties associated with rockfall hazards following a Bayesian approach is provided in Chapter 5.

A rational and consistent understanding and consideration of uncertainties is vital for any risk assessment and risk management process and for ensuring rational and optimal decision making. It is hence useful and instructive to think about the nature of the various types of uncertainties, particularly in the context of risk communication. While communicating the results of risk assessments and analyses with the outside world, it is important to distinguish primarily between the objective probabilities related to scatter and uncertainty from a natural origin on the one hand and subjective probability estimates for knowledge (epistemic) uncertainties on the other hand. When considering updating and incorporation of new knowledge, it is important to understand how uncertainties change characteristics as functions of both the point in time where they are looked upon and as functions of the scale of the modeling used to represent them. This also influences the level of detail required for the treatment of uncertainties in any risk assessment and risk management process.

# 7 REFERENCES

Aitchison, J. And Dunsmore, I.R. (1975). Statistical Prediction Analysis. Cambridge University Press, Cambridge.

Apel H, Thieken AH, Merz B, Blöschl G (2004). Flood risk assessment and associated uncertainty. Natural Hazards and Earth System Sciences, 4:295-308

Apostolakis, G. (1990). The Concept of Probability in Safety Assessment of Technical Systems. Science, New Series, 250(4986): 1359-1364.

Azzoni, A., La Barbera, G. & Zaninetti, A. 1995. Analysis and prediction of rockfalls using a mathematical model. International Journal of Rock Mechanics and Mining Science & Geomechanics Abstracts, 32(7), pp. 709-724.

Baillifard F., Jaboyedoff M., Sartori M. (2003). Rockfall hazard mapping along a mountainous road in Switzerland using a GIS-based parameter rating approach. Natural Hazards and Earth System Sciences, 3, pp. 431–438.

Bayraktarli, Y.Y., Ulfkjaer, J.P., Yazgan, U. and Faber, M.H. (2005). On the Application of Bayesian Probabilistic Networks for Earthquake Risk Management. In: Augusti et al., eds. Safety and Reliability of Engineering Systems and Structures. Proceedings of the ICOSSAR 2005, Rome. Rotterdam: Millpress, 3505-3512.

Bayraktarli, Y.Y., Yazgan, U., Dazio, A. and Faber, M.H. (2006). Capabilities of the Bayesian probabilistic networks approach for earthquake risk management. In: Proceedings of ECEEs First European Conference on Earthquake Engineering and Seismology, September 3-8, Geneva, Switzerland.

Benjamin, J.R. and Cornell, C.A. (1970). Probability, Statistics and Decision for Civil Engineers. McGraw-Hill, NY.

Budetta P. (2004). Assessment of rockfall risks along roads. Natural Hazards and Earth System Sciences, 4(1), pp. 71-81.

Chau, K.T., Wong, R.H.C. & Wu, J.J. 2002. Coefficient of restitution and rotational motions of rockfall impacts. International Journal of Rock Mechanics and Mining Sciences, 39(1), pp. 69-77.

Coles S. (2001). An Introduction to Statistical Modelling of Extreme Values. Springer, London.

Coles S., Pericchi L.R., Sisson S. (2003). A fully probabilistic approach to extreme rainfall modelling. Journal of Hydrology, 273(1), pp. 35-50.

Dorren, L.K.A. (2003), A review of rockfall mechanics and modelling approach, Progress in Physical Geography 27(1): 69-87.

Dussauge-Peisser, C., Helmstetter, A., Grasso, et al. (2002). Probabilistic approach to rock fall hazard assessment: potential of historical data analysis, Natural Hazards and Earth System Sciences, 2, pp. 15-26.

Duzgun H.S.B., Yucemen M.S., Karpuz, C., (2003). A Methodology for Reliability-Based Design of Rock Slopes. Rock Mechanics and Rock Engineering, 36(2), pp. 95-120.

Ettlin, A. 2006. Rigid body dynamics simulation for robot motion planning. Doctoral Thesis, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland.

Faber, M.H. (2002). Risk Based Inspection: the Framework. Structural Engineering International. 12(3): 186-195.

Faber, M.H. and Stewart, M.G. (2003). Risk Assessment for Civil Engineering Facilities: Critical Overview and Discussion. Reliability Engineering and Systems Safety, 80(2): 173-184.

Faber, M.H., Ranamukhaarachchi, S.L. and Rego, L. (2005). Sustainable Management of Natural Hazards in the Region of South-East Asia and South Asia. In: Proceedings of the International Symposium on Disaster Reduction on Coasts. November 14-16, 2005, Melbourne, Australia.

Faber, M.H., Bayraktarli, Y.Y. and Nishijima, K. (2007). Recent developments in the management of risks due to large scale natural hazards. In: Proceedings of SMIS XVI. Mexican National Conference on Earthquake Engineering. November 1-4, 2007, Ixtapa, Guerrero, Mexico.

Graf, M. et al., Nishijima, K. and Faber, M.H. (2009). A probabilistic typhoon model for the Northwest Pacific Region. In: Proceedings of the 7th Asia Pacific conference on Wind Engineering. November 8-12, Taiwan.

Guzzetti F., Crosta G., Detti R., Agliardi F. (2002). STONE: a computer program for the three-dimensional simulation of rock-falls. Computers and Geosciences, 28(9), pp. 1079-1093.

Hall J.W. (2003). Handling uncertainty in the hydroinformatic process. Journal of Hydroinformatics, 5(4), pp. 215-231.

Helton J.C. and Burmaster D.E. (1996). Guest editorial: treatment of aleatory and epistemic uncertainty in performance assessments for complex systems. Reliability Engineering & System Safety, 54(2-3), pp. 91-94.

Hungr, O. Evans, S. G., and Hazzard, J., (1999). Magnitude and frequency of rock falls and rock slides along the main transportation corridors of southwestern British Columbia, Can. Geotech. J., 36, pp. 224-238.

JCSS (2000). Guideline for Reliability Based Assessment of Existing Structures. The Joint Committee on Structural Safety, Rilem.

JCSS (2001). Probabilistic Model Code for Reliability Based Design. The Joint Committee on Structural Safety.

JCSS (2008). Risk Assessment in Engineering. The Joint Committee on Structural Safety. Internet Publication: http://www.jcss.ethz.ch/publications/JCSS_RiskAssessment.pdf

Jensen F.V. (2001). Bayesian Networks and Decision Graphs. Springer, New York.

Jimenez-Rodriguez R., Sitar N., Chacón J. (2006). System reliability approach to rock slope stability, International Journal of Rock Mechanics and Mining Sciences, 43(6), pp. 847-859.

Lindley, D.V. (1976). Introduction to Probability and Statistics from a Bayesian Viewpoint. Cambridge, Cambridge University Press.

Madsen, H.O., Krenk, S. and Lind, N.C. (1986). Methods of Structural Safety. Prentice-Hall Inc., Englewood Cliffs, NJ.

Mann C.J. (1993). Uncertainty in geology. In: Davis J.C., Herzfeld U.C. (eds.), Computers in geology – 25 years of progress, Oxford Univ. Press, p. 241–254.

McGuire, R.K., Cornell, C.A., and Toro, G.R. (2005). The case for using mean seismic hazard. Earthquake Spectra 21, pp. 879-886; doi:10.1193/1.1985447.

Nash, C. (2003). Unification of Accounts and Marginal Costs for Transportation Efficiency, UNITE. Final Report, 5th Framework RTD Programme, Leeds.

Nishijima, K. and Faber, M.H. (2007). A Bayesian framework for typhoon risk management. In: Proceedings of the 12th International Conference on Wind Engineering. July 1-6 2007, Cairns, Australia. pp. 1959-1966.

Paté-Cornell M. E. (1996) - Uncertainties in risk analysis: Six levels of treatment. Reliability Engineering and System Safety (54): 95-111.

Raiffa, H. (1968). Decision Analysis, Introductory Lectures on Choice under Uncertainty, Addison-Wesley, Reading, Mass.

Raiffa, H. and Schlaifer, R. (1961). Applied Statistical Decision Theory. Harvard University Press, Cambridge University Press, Cambridge, Mass.

Robotham, M. E., Wang, H., and Walton, G. (1995). Assessment of Risk from Rockfall from Active and Abandoned Quarry Slopes. Transactions of the Institution of Mining and Metallurgy Section A-Mining Industry, 104, A25-A33.

SafeLand (2011). Dealing with uncertainties in modelling, prediction, and decision-making. Deliverable D0.3 report, SafeLand: Seventh Framework Programme for research and technological development (FP7) of the European Commission.

Savage, L. J. (1954). The Foundations of Statistics, John Wiley & Sons, Inc., NY.

Schubert M., Straub D., Faber M.H. (2005). Reliability of rock fall protection galleries – A case study with a special focus on the uncertainty modelling. Safety and Reliability of Engineering Systems and Structures (Proc. ICOSSAR 05, Rome), Augusti et al. (eds), Millpress, pp. 1333-1340.

Stevens W.D. (1998). RocFall: A tool for probabilistic analysis, design of remedial measures and prediction of rockfalls. Report, Department of Civil Engineering, University of Toronto, Canada.

Straub, D. (2005). Natural hazards risk assessment using Bayesian networks. In: Augusti et al., eds. Safety and Reliability of Engineering Systems and Structures. Proceedings of the ICOSSAR 2005, Rome. Rotterdam: Millpress, 2535-2542.

Straub D. and Der Kiureghian A. (2008). Improved Seismic Fragility Modelling from Empirical Data. Structural Safety, 30(4), pp. 320-366.

Straub, D. and Schubert, M. (2008). Modeling and managing uncertainties in rock-fall hazards. Georisk, 2(1): 1-15.